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THESIS

MILITARY MANPOWER PLANNING: OPTIMIZATION MODELING FOR THE ARMY OFFICER ACCESSION/BRANCH DETAIL PROGRAM

by

Jeffrey C. Corbett

September 1995

Thesis Advisor: James R. Wood III

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OA/BDM is a multi-year weighted goal program designed to maximize the Army's ability to meet forecasted authorization requirements subject to OPMD policy guidance. This study demonstrates that multi-year goal programs such as OA/BDM serve as meaningful analytical tools and that the dynamic capability of these models offer benefits that steady state models cannot provide. Additionally, feedback derived from OA/BDM and queuing theory suggest that the current two and four year Army detail plan does not offer a viable means for aligning lieutenant overages among Army career branches.

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MILITARY MANPOWER PLANNING: OPTIMIZATION MODELING FOR THE ARMY OFFICER ACCESSION/BRANCH DETAIL PROGRAM

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Submitted in partial fulfillment
of the requirements for the degree of

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ABSTRACT

The purpose of this thesis is to provide the Army's Officer Personnel Management Directorate (OPMD) with a flexible, responsive, manpower optimization model that assists personnel planners in determining yearly officer accessions and serves as an analysis tool with which to evaluate the impact of planned accessions. This thesis also surveys the Army's Branch Detail Program and its impact on the problem of balancing the lieutenant overages that occur among the Army's career branches. The modeling effort put forth in this study combined multiobjective programming, probability theory, and insights gained from queuing theory to develop a multiyear manpower planning model known as the *Officer Accession Branch Detail Model (OA/BDM)*.

OA/BDM is a multi-year weighted goal program designed to maximize the Army's ability to meet forecasted authorization requirements subject to OPMD policy guidance. This study demonstrates that multi-year goal programs such as OA/BDM serve as meaningful analytical tools and that the dynamic capability of these models offer benefits that steady state models cannot provide. Additionally, feedback derived from OA/BDM and queuing theory suggest that the current two and four year Army detail plan does not offer a viable means for aligning lieutenant overages among Army career branches.

THESIS DISCLAIMER

The views expressed in this thesis are those of the author and do not reflect the official policy or position of the Department of Defense or the U.S. Government. Additionally, the reader is cautioned that the computer program developed in this research may not have been exercised for all cases of interest. While every effort has been made, within the time available, to ensure that the program is free of computational and logic errors, they cannot be considered validated. Any application of this program without additional verification is at the risk of the user.

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EXECUTIVE SUMMARY

The Army's Officer Personnel Management Directorate (OPMD) procures the number of new lieutenants required in each career branch to meet future officer manpower requirements. The business of procuring and allocating officers to meet future branch needs, combined with budgetary constraints and the nature of the Army branch and grade structure, presents special challenges to OPMD.

The purpose of this thesis is to provide Army personnel planners with a flexible and responsive manpower model that assists in determining yearly officer accessions and serves as an analysis tool for evaluating the impact of planned accessions. This thesis also surveys the Branch Detail Program -- a process of sharing excess lieutenants among branches--and its impact on the problem of balancing the lieutenant overages that occur among the Army's career branches.

The modeling effort put forth in this study develops a manpower planning model known as the *Officer Accession/Branch Detail Model (OA/BDM)*. *OA/BDM* is a model designed to maximize the Army's ability to meet current and future authorization requirements while satisfying OPMD policy objectives. Subject to OPMD planning guidance, the model works by aging an initial inventory over a specified planning horizon and determining officer accessions based upon the ability of projected inventories to meet future requirements.

To demonstrate OA/BDM and to gain a measure of its effectiveness as an analysis tool, this study uses the model to implement predetermined accession plans derived from a model currently used by officer accession planners. The feedback generated by OPMD's accession model is compared to recommendations made by OA/BDM to determine how well each accession plan meets authorization targets and OPMD accession objectives.

The results of the demonstration indicate that accession recommendations produced by OA/BDM show potential for improving OPMD's ability to forecast future officer inventories and consequently, better satisfy officer accession objectives. The results also indicate that assumptions inherent in OPMD's model and its resulting accession plans tend to overestimate the Army's ability to meet officer manpower requirements. Additionally, feedback from OA/BDM suggests the current program of loaning excess lieutenants for two and four years does not offer a viable means for balancing lieutenant overages. Further analysis performed using relationships derived and presented in this study, support the findings of OA/BDM and suggest that standard three or four year programs, or a combination thereof, offer a more tractable means for achieving balance among lieutenants in career branches.

In summary, this study demonstrates that OA/BDM can serve as a meaningful analytical tool for Army officer accession planners and that its dynamic capabilities offer benefits to the Officer Personnel Management Directorate that their current model may not provide. Furthermore, the model's flexibility shows that it is readily adaptable to a host of different purposes related to manpower and force structure planning.

I. INTRODUCTION

The Army's Officer Personnel Management Directorate (OPMD) manages the careers of more than 103,000 warrant and commissioned officers. The directorate manages all Army officers except general officers and those in the legal, and chaplain specialties. Among their many responsibilities, OPMD procures the number of new lieutenants required in each career branch to maximize the Army's ability to meet current and future officer manpower requirements. The business of procuring enough officers to meet future needs, combined with the dynamics of budgetary constraints and the nature of the Army branch and grade structure, presents a special challenge to OPMD known as the *Officer Accession/Branch Detail Problem*. This thesis describes a manpower planning optimization approach and its application to the Officer Accession/Branch Detail problem. The purpose of this study is to assist OPMD in determining the yearly allocation of newly commissioned officers to bring into each career branch, and to provide them with an analysis tool with which to study the impact of planned accessions.

A. BACKGROUND

The Army traditionally categorizes its core structure into three areas -- Combat Arms (CA), Combat Support (CS) and Combat Service Support (CSS). Each area is comprised of a set of branches with unique authorization structures, designed to support the Total Army in its execution of the National Military Strategy. Even though branch structures vary with time, they historically exhibit certain characteristics. In general, combat arms branches have a high demand for junior officers, while CS and CSS branch structures share smaller demands for junior officers. In contrast, CS and CSS branches have a high demand for senior grade officers, while CA branches exhibit a lower demand for senior officers. However, all branches share one characteristic. Each branch has a great demand for mid-grade officers -- captains. Furthermore, the gap between the demand for junior and mid-grade officers is generally small in CA, and large in the CS and CSS branches. These characteristics present challenges to OPMD, as they manage the personnel inventory to meet the specific and often conflicting requirements of each branch. Throughout recent history, Army manpower planners have used many management tools, such as voluntary and involuntary transfers to alleviate structural disparities that exist between grades and branches. One such program was called Force Alignment Plan III (FAP III).

1. Force Alignment Plan III

In March 1984, the Army Chief of Staff approved FAP III as a realignment program to help smooth out structural differences that existed between grades in various branches of the Army. Under FAP III, a centralized board adjusted the officer inventory by transferring or "re-branching" Other Than Regular Army Officers (OTRA), selected for promotion to captain (OPD - P, 1987). Typically, personnel managers used large pools of excess combat arms lieutenants to fill projected shortages for CS and CSS captains. Thus, as necessary, realignment boards transferred OTRA combat arms lieutenants into CS and CSS branches. Once selected, newly promoted OTRA officers left their initial tours of duty to attend officer advanced schooling to prepare themselves for assignments in their new branches.

2. The Branch Detail Program FY '86

The realignment process of FAP III quickly became unpopular, as many lieutenants were transferred against their desires, and against the wishes of their commanders. Thus, in November of 1986, the Army Deputy Chief of Staff for Personnel (DCSPER) approved the Branch Detail Program. The principal purpose of this program was to permit inventory alignment without the unpopular forced rebranching of FAP III. The program required that OPMD identify a sufficient amount of new lieutenants to transfer in advance, to meet projected requirements when officers reached the promotion point to captain. While FAP III applied only to OTRA officers, the Branch Detail Program was opened to both Regular Army (RA), and OTRA officers. This meant that selected lieutenants would serve in the combat arms for their initial four years of service, and then return to their CS or CSS branch for the remainder of their time in service. In practice, once selected for promotion, detailed officers left their initial combat arms assignment to gain on-the-job experience in their new branch, or they immediately departed for officer advanced schooling to prepare for their new assignments. (OPD-P, Shupack,1987, 1989)

3. The Branch Detail Program FY '89

In 1989, the DSCPER, modified the program from a standard four year plan to a four and a two year detail program. This program is the Branch Detail Program still in use today. However, the focus of the program has evolved to be more in line with the benefits that resulted from its use, rather than as a tool solely used to realign the force structure at the captain promotion point. Today, the Branch Detail Program is viewed as serving several very important purposes. Most importantly, from a leadership standpoint, the mission of the program is to maximize the combat arms experience of the entire officer corps, as well as to increase the availability of trained combat arms lieutenants. At the same time, the Branch Detail Program continues to serve as a proactive management tool for aligning the force structure.

B. OFFICER ACCESSIONS/BRANCH DETAIL

1. Basic Branch Accessions

Each year the DCSPER determines the *accession cohort* -- the total number of lieutenants to be commissioned into active duty during a fiscal year. OPMD distributes the accession cohort by accessing lieutenants into each of fifteen basic branches. For study purposes, these branches are categorized as either *combat* or *non-combat arms*. Subject to planning guidance, the Distribution Division of OPMD recommends the allocation of lieutenants to be accessed into each branch. OPMD refers to the number of lieutenants to access into each branch to meet future requirements as *core accessions*. Because long term grade requirements are, on the whole, greater than near term grade requirements, the Army accesses excess lieutenants into each branch. However, because of budgetary constraints, the accession cohort does not allow OPMD to access enough lieutenants to meet all peacetime grade requirements.

2. Control Branch Accessions

As stated above, the number of officers that are accessed exceed all branch needs for lieutenants. This is particularly pronounced for the non-combat arms. Consequently, from a force alignment standpoint, a need arises that OPMD distribute this excess equitably across all branches. Thus, OPMD designates a portion of the excess non-combat arms officers as *branch detailed officers*. Although core accessed into a non-combat arms branch, the branch detailed officers are loaned to the combat arms branches where they serve for up to four years before returning to their non-combat arms basic branch. OPMD refers to the number of lieutenants to access into each branch such that lieutenant overages balance as *control branch accessions*. *Control branch accessions* are defined as *core accessions* plus or minus *branch detailed officers*.

Currently, the Military Intelligence (MI), Adjutant General Corps (AG), Signal Corps (SC), Finance (FI), Transportation (TC), Ordinance (OD), and Quartermaster Corps (QM), loan lieutenants to serve in the Infantry (IN), Armor (AR), Field Artillery (FA), Air Defense Artillery (ADA), and Chemical Corps (CM). OPMD refers to branches that loan excess lieutenants as *donor branches*, and branches that receive excess lieutenants as *receiver branches*. MI and AG loan lieutenants to receiver branches for four years. The other donor branches loan their lieutenants for two years.[1]

---

[1] Note that CM is not considered a combat arm, but because of its similarities in grade structure, it is added to the set of receiver branches.

3. Current Accession Policy

OPMD has specified several objectives that must be considered when determining accessions. The principal mission is to meet as close as possible the demand for accessions in each branch, subject to the following policies: 1) meet each branch's total officer authorization requirements; 2) access enough lieutenants to meet the combat arms captain requirements; 3) access enough lieutenants to meet, as best as possible, grade requirements in each branch; 4) distribute the entire accession cohort; 5) do not let accessions vary greatly from year to year; and 6) balance lieutenant overages.

C. CURRENT ACCESSION MODEL & CONCERNS

Although the current officer accession/branch detail model produces satisfying recommendations for core and control branch accessions, there are several factors and assumptions that are concerns: 1) the current model does not use current inventory data; thus, past accessions, and separations, do not influence current accessions; 2) the model implicitly assumes that past and future accessions are constant; and 3) the model does not consider branch specific retention patterns. More importantly however, OPMD wishes to study the impact of officer accessions and branch detail policy on the distribution of officers in the combat and non-combat arms. Currently, the Distribution Division of OPMD does not have a robust model that will allow them to regularly conduct this type of analysis. The current officer accession model cannot be used to conduct sensitivity analysis on a range of questions. For instance, how aligned will the branches be in the year 2000 if accessions decrease yearly by a given percent? Given anticipated yearly accessions and authorizations, how well can the Army meet needs for combat support captains? Given this year's accessions, how well will each branch be able to meet projected authorizations in the year 2000?

D. STUDY SCOPE

The scope of this study involves developing a flexible, responsive, manpower planning optimization model to assist personnel planners in maximizing the Army's ability to meet branch specific officer strength requirements, subject to the goals and objectives of the Deputy Chief of Staff for Personnel, and the Officer Personnel Management Directorate. The goal is to provide a model that serves as a guide for determining the yearly allocation of core and control branch accessions, and allows OPMD to investigate the long term implications of annual officer accessions. Furthermore, the study examines the current two and four year detail program and its impact on the goal to balance lieutenant overages. This study includes a demonstration of the model using data provided by the Distribution Division of OPMD.

E. CHAPTER OUTLINE

Chapter II presents a review of manpower modeling approaches and their application to military manpower systems. It also presents the approach this study uses to model the Officer Accession/Branch Detail Problem. Chapter III presents a multi-year goal programming formulation for determining core accessions. Chapter IV presents an analysis of the current branch detail policy and extends the model presented in Chapter III to include determining control branch accessions. Chapter V presents a demonstration of the model and analysis of results using data provided by the Distribution Division OPMD. Finally, Chapter VI offers conclusions and recommendations for further study.

II. MILITARY MANPOWER MODELING

The basic manpower planning problem is to determine the number of people with various skills to best meet future requirements. When modeling large personnel systems, it is inefficient to track personnel inventories by monitoring each individual. Thus, most manpower models aggregate individuals by class descriptors. For example, in the Army personnel system, class descriptors might include year group, branch, functional area, grade, and years of commissioned service, just to name a few. Each individual in the inventory can be a member of only one class, but can change classes based upon transition assumptions. For example, in order to distinguish personnel in each class, models might use the following notation:

$Y(b,k,t) \equiv$ the number of personnel in branch b, with k years of service in time t.

The combination (b, k, t) is called a *state*. An individual can be in only one state in any given period. Some of the basic manpower modeling approaches which manage personnel inventories using state variables include: Transition Rate (Markov Models), Network Flow, and Goal Programming models (Gass, 1990). The literature is full of examples of how these approaches can be applied to manpower planning. Grinold and Marshall (1977), Klingman and Phillips (1984), and Charnes and Cooper et al. (1977) present the theory and the mathematics of these approaches as well as many applications to manpower modeling. This chapter describes Goal Programming and its application to manpower planning. Furthermore, based upon these concepts, this chapter presents an approach to modeling the Officer Accession/Branch Detail problem.

A. GOAL PROGRAMMING

Goal programming is one of the oldest and most widely used approaches in mathematical modeling. First introduced in the 1950's, by Charnes and Cooper et al., its overall purpose is the satisfaction of multiple objectives being considered in the same problem context. The goal program minimizes deviations between the achievement of a decision maker's objectives, and the desired level of achievement for that objective. (Romero, 1991)

There are many goal programming variants. Presented here is one of the most widely used variants called the Weighted Goal Program (WGP). The basic form of the WGP is:

$$\begin{aligned} &\mathit{Minimize} \sum_{i=1}^{k} (\alpha_i n_i + \beta_i p_i) \\ &s.t. \quad f_i(x) + n_i - p_i = tgt_i \qquad (2.1) \\ &\boldsymbol{x} \in \boldsymbol{F} \\ &\boldsymbol{x} \geq \mathbf{0} \quad \boldsymbol{n} \geq \mathbf{0} \quad \boldsymbol{p} \geq \mathbf{0} \end{aligned}$$

where $f_i(x)$ represents a set of objectives; tgt_i represents a target value, or desired level of attainment for the *ith* objective; n_i and p_i represent the absolute negative and positive deviations from the attainment of the *ith* objective; and $\boldsymbol{F}$ represents the traditional feasibility constraint set, often referred to as hard constraints. The introduction of the deviational variables, n_i and p_i, with the objective, $f_i(x)$, and its desired attainment level, tgt_i, is referred to as a goal constraint (Romero, 1991). The objective function is a weighted composite of the goals, where α_i and β_i represent the weight that a decision maker attributes to negative and positive deviations from the *ith* objective. Thus, a WGP is comprised of a weighted objective function, goal constraints, and hard constraints. The goal constraints may or may not be fully achievable; whereas, hard constraints must be achieved in order for a solution to be feasible. By minimizing the weighted deviational variables associated with each goal, the WGP in effect achieves all objectives simultaneously. (Romero, 1991) Thus, for a given set of weights, the WGP produces a "satisficing", rather than an optimal solution for a given set of objectives.[2]

B. GOAL PROGRAMMING & MANPOWER PLANNING MODELS

The appeal of the goal program is in its ability to satisfy numerous objectives in the same problem context. This is particularly appealing to decision makers as trying to attain several objectives at once mirrors the reality of decision making more closely than the traditional single optimization model. Given its appeal, there are many examples of its use in military manpower planning. Price and Piskor (1972) use a goal programming formulation to model the manpower system for officers in the Canadian Forces. Bres and Burns et al. (1980) use goal programming to determine officer accessions from various commissioning sources for the United States Navy (1980). Finally, Gass, (1982), applies GP to determine separation, recruitment, and promotions for the United States Army Enlisted Force. An interesting and powerful aspect of the three aforementioned applications is that their authors extend the basic concept previously described by adding a time dimension to the formulation. The basic form of a multiyear weighted goal program is:

---

[2] Satisficing is a term originated by Herbert Simon that coins the words *satisfactory* and *optimizing*. It refers to the tendency of decision makers to seek solutions that are not entirely "optimal" in the mathematical or economic sense, but satisfying given the reality of the circumstances at hand. (Hillier, 1990)

$$\begin{aligned} &\mathit{Minimize} \quad \sum_{t=1}^{T} \sum_{i=1}^{k} (\alpha_i(t)\, n_i(t) + \beta_i(t)\, p_i(t)) \\ &s.t \quad f_i(x(t)) + n_i(t) - p_i(t) = tgt_i(t) \qquad (2.2) \\ &\boldsymbol{x}(t) \in \boldsymbol{F}(t) \\ &\boldsymbol{x} \geq \boldsymbol{0} \;\; \boldsymbol{n} \geq \boldsymbol{0} \;\; \boldsymbol{p} \geq \boldsymbol{0} \end{aligned}$$

The formulation represents the desire to minimize the deviation from attainment of the *ith* objective in period t. The deviational variables, $n_i(t)$ and $p_i(t)$, represent the absolute deviation from attainment of the target, tgt_i, in time t; $\alpha_i(t)$ and $\beta_i(t)$ represent penalty weights associated with deviations from the desired target in time t; and, $\boldsymbol{F(t)}$ represents the feasible constraint set for time t. In manpower planning models, the vector, $\boldsymbol{x(t)}$, would include state variables. For example, a variable, $x_{bs}(t)$, might represent the number of personnel in branch b with skill s in time t. Large scale manpower planning models often include many state variables and many different goal constraints. In practice, these mulityear planning models have become very powerful planning and policy analysis tools for the decision maker.

Despite the appeal of the WGP, there are several criticisms. A basic assumption of the WGP is that trade-offs between the attainment of goals are constant. This assumption violates principles set forth in economic theory which establish that trade-offs are not linear

(Rosenthal, 1983). Furthermore, the weights in a WGP represent a decision maker's implicit and explicit priorities toward achievement of objectives. Thus, as alluded to earlier, there is no true optimal solution, but only a satisficing solution that is acceptable to the decision maker. Consequently, the WGP solution often revolves around the establishment of an appropriate set of weights (Gass, 1986).

C. METHODOLOGY

The core of this study is to present a modeling approach to the Officer Accession/ Branch Detail problem. This study operates under the assumption that an officer that is loaned to another branch survives at the same rate as officers that are never loaned. The implication here is that the loaning of officers does not impact on a branch's ability to meet future requirements beyond the grade of lieutenant. If branch detailed officers survive the same as all other officers, this would be the case, since all detailed officers return to their basic branch before reaching choke points that exist only after the fourth year of service.

Given this assumption, this study approaches the Officer Accession/Branch Detail problem in two phases. First, core accessions are modeled using a multiyear weighted goal program. Next, the issue of balancing lieutenant overages is examined using Little's Law, a result from stochastic queuing theory. Then, employing results derived from queuing theory, the study extends the core accession model to include control branch accessions -- balancing lieutenant overages.

III. DETERMINING BASIC BRANCH ACCESSIONS

This chapter presents a multiyear goal programming formulation for determining core accessions--the Officer Accession Model (OAM). The chapter begins with a summary of OAM, followed by a description of how the model works. Next, the model formulation is presented in algebraic form, with explanation. The chapter concludes with a discussion of how weights are determined for the model.

A. SUMMARY OF OAM

OAM determines core accessions for each year in a planning horizon, while satisfying OPMD officer accession policy. That is, the model fulfills the demand for accessions in each branch, subject to the following: 1) meet each branch's total officer authorization requirements; 2) access enough lieutenants to meet the combat arms captain requirements; 3) access enough lieutenants to meet, as best as possible, grade requirements in each branch; 4) distribute the entire accession cohort; 5) do not let the total inventory exceed specified limits; and 6) do not let accessions vary greatly from year to year.

In short, OAM is comprised of a weighted composite objective function that minimizes the negative deviation from policy objectives; a set of goal constraints; a set of hard constraints, and a set of boundary conditions. The set of goal constraints include equations that strive to fulfill: a) the objective to meet the total officer requirements for each branch; and b) the objective to meet target authorizations in each branch and grade. The set of hard constraints include equations that ensure: a) old and new inventory is aged, and new accessions determined; b) the entire accession cohort is distributed; and c) the total officer inventory limit is not exceeded. Bounds on accessions ensure: a) changes in accessions are limited from year to year, and b) core accessions into each branch do not fall below or above established minimum and maximum amounts.

B. OAM: HOW IT WORKS

OAM works by aging an initial inventory over a specified planning horizon, and determining core accessions for each year based upon the available inventory of officers to fill a set of authorization targets. The model ages inventory using conditional and unconditional probabilities. For example, Figure 1 represents the survival function for the

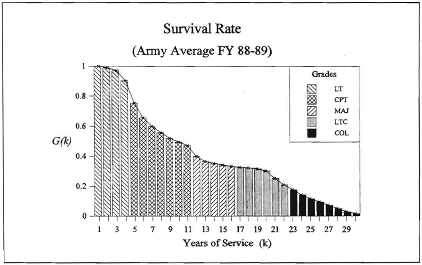

Figure 1. The Survivor Function

lifetime of an army officer, denoted *G*(*k*). If *K* is the number of years an officer remains in service, then *G*(*k*) is equal to *P*[*K*>*k*] -- the unconditional probability that an officer survives more than *k* years of commissioned service. Although represented as a set of probabilities, *G*(*k*) also represents the fraction of a cohort of officers that remain in service *k* years after they enter. Given *G*(*k*), the conditional probability that officers in their *kth* year of commissioned service survive an additional *y* years is:

$$P[K>k+y|K>k] = \frac{P[K>k+y, K>k]}{P[K>k]} = \frac{P[K>k+y]}{P[K>k]} = \frac{G(k+y)}{G(k)} \qquad (3.1)$$

Thus, using Figure 1, the probability that an officer in year five survives to year eight is *P*[*K*>8]/P[*K*>5] = 0.56÷0.75 = 0.69. That is, sixty-nine percent of the officers in their fifth year of service survive their eighth year of service. Thus, given *G*(*k*), OAM calculates conditional survival probabilities. Using the derived set of probabilities, OAM ages the current inventory, and uses that aged inventory to determine subsequent accessions.

For example, Figure 2 represents an initial inventory of infantry officers for FY '95. The line superimposed on the graph represents the theoretical inventory that would exist if the YG '95 cohort survived at the rates shown by the survivor function in Figure 1. A trace of the inventory stacks, shown in Figure 2, would produce an empirical distribution that crudely resembles the theoretical distribution -- the line-- shown in Figure 2. The shaded stacks represent the inventory of officers who have achieved the grade level indicated. Figure 3 represents the initial inventory shown in Figure 2, aged forward in time by four years.

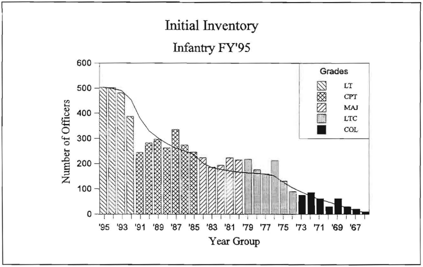

Figure 2. Infantry FY'95 Inventory

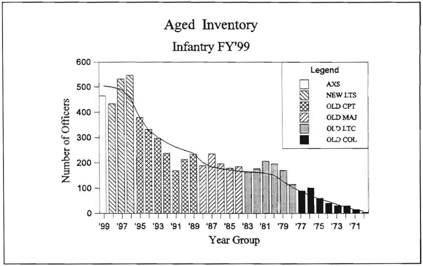

Figure 3. Aged '95 Inventory & Subsequent Accessions

Shown as "old" is the remainder of the initial FY '95 inventory that survived four years. The old inventory now comprises the grade categories indicated. Shown as "new", are the officers that OAM recommended accessing in years '96, '97, and '98 that survived three, two, and one years, respectively. Finally, the stack labeled '99 represents accessions for the fourth year -- the end of the planning horizon.

For each branch, and in each year, OAM aggregates subsets of the inventory that typically represent the officers that comprise a particular grade category. By comparing these aggregations to authorized strength targets for each branch and grade, the model determines accessions such that authorization targets and OPMD policies are met.

C. MODEL FORMULATION: OAM

We present an algebraic formulation of OAM following an introduction to notation. Model implementation uses the General Algebraic Modeling System (GAMS ver. 2.25).

1. Indices

b	Branches	*IN, AR, FA, AD, CM, SC, FI ,TC, OD, QM, MI, AG, AV, EN, MP*
r	Combat Branches	*IN, AR, FA, AD, CM*
d	Support Branches	*SC, FI, TC, OD, QM, MI, AG, AV, EN, MP*
g	Grades	*LT, CPT, MAJ, LTC, COL*
k	Year of Service	$1, 2, 3, 4,\ldots,K$ ***where*** K ***denotes max service length***
t	Planning Year	$1, 2, 3, 4,\ldots, T$ ***where*** $T < K$

2. Derived Sets

$l(g)$	Lower limit for years of commissioned service in grade g
$u(g)$	Upper limit for years of commissioned service in grade g

3. Data

The data divides into the following groups: problem initialization data, derived data, authorization targets, and objective function drivers. The derived data are not inputs, but are calculated using inputs during the run time of the model.

a. Initialization Data

$I_{0b}(k)$	Initial inventory of officers in branch b in their *kth* year of commissioned service (YCS)
$LYAXS(b)$	Previous year's accessions in branch b
S_{bk}	Survival rate -- the unconditional probability an officer in branch b survives the *kth* year of commissioned service
Low, Up	Max proportions of change in accessions between years
r	Discount factor for follow on objective function weights
$min_b(t)$	Minimum number of accessions into branch b in year t
$max_b(t)$	Maximum number of accessions into branch b in year t

b. Derived Data

$S_{bt}(k)$	The conditional probability that officers in branch b in their *kth* year of commissioned service (YCS) survive year t
$\delta(t)$	The discount rate for year t

c. *Authorization Data*

$AU_{bg}(t)$	Authorization target for officers in branch b, for grade g, in year t
$TOTAL_b(t)$	Total authorization target for officers in branch b in year t
$ITOTAL(t)$	An upper limit for the total officer inventory in year t
$ACCOH(t)$	The accession cohort from DCSPER -- the number of officers to be commissioned in year t

d. *Objective Function Penalty Coefficients*

$w_{bg}(t)$	Weight for negative deviation from targets in branch b, grade g, time t
$bpen_b(t)$	Penalty weight for missing total authorization target in branch b

4. Variable Definitions

The variables are categorized as decision variables, and auxiliary variables. The auxiliary variables are positive variables that capture the absolute positive and negative deviation from objective targets.

a. *Decision Variables*

$y_{bk}(t)$	Inventory of officers in branch b in their *kth* year of commissioned service in year t
$x_b(t)$	Accessions to branch b in year t

b. *Auxiliary Variables*

$under_{bg}(t)$	Inventory shortfall for branch b, grade g, time t
$surplus_{bg}(t)$	Inventory surplus for branch b, grade g, time t

$bunder_b(t)$	Shortage from branch authorization in b in year t
$bover_b(t)$	Number of officers over branch authorization in b, year t

5. The Objective Function

OAM enforces three objectives in the composite objective function (Equation 3.2). The function minimizes over the planning horizon the weighted deviations from authorization targets. The goals are expressed in the objective function in their order of priority with respect to the accession policy. They are: a) meet the total officer requirements in each branch; b) meet the combat arms captain requirements; and c) meet target authorizations in each branch and grade. Although not explicitly shown below, the second priority is enforced by assigning a higher penalty weight for combat arms captains than for all other branch and grade requirements.

$$Min \sum_{t}^{T} \delta(t) \left[\sum_{b} bpen_{b}(t)\ bunder_{b}(t) + \sum_{b} \sum_{g} w_{bg} under_{bg}(t) \right] \qquad (3.2)$$

Because future OPMD policy and authorizations are subject to change, the long term impacts of these factors on near term accessions are reduced by discounting the objective function over the planning horizon. This is accomplished using a discount function, $\delta(t)$. The function $\delta(t)$ represents the *present discounted value* of one penalty unit accessed in year t. The discount function is:

$$\delta(t) = \frac{1}{(1+r)^t} \qquad (3.3)$$

where r is a discount rate chosen by the decision maker. (Nicholson, 1992)

In OAM, the composite objective function only minimizes the deviation variable of interest. For example, since OPMD policy dictates accessing at a minimum, enough officers to meet total authorization requirements in each branch, the objective function minimizes only the negative deviational variable associated with this goal. Likewise, the objective function minimizes only negative deviations from branch and grade targets. This is done so that, given a set of weights, the optimization has the maximum amount of freedom in posturing accessions to best meet grade requirements in each branch.

As stated earlier, weights are an important aspect to any WGP. As such, the details of how weights were determined are discussed in the next section. For now, let it suffice to say that the weights represent the relative demand to meet a particular branch and grade target.

6. Goal Constraints

Equation 3.4 represents the goal to meet each branch's total officer authorization requirement:

$$\sum_k y_{bk}(t) + bunder_b(t) - bover_b(t) = TOTAL_b(t) \quad for\,all \;\; b,\ t \qquad (3.4)$$

For each year, and for each branch, this equation aggregates the total number of officers in a branch by summing over *k*, the state variable, $y_{bk}(t)$. The equation compares this aggregation, which represents the total number of officers in a branch in year *t*, to the branch's total officer authorization target, $TOTAL_b(t)$. The auxiliary variables, $bunder_b(t)$ and $bover_b(t)$, measure the absolute deviation from attainment of the target in each branch for each year.

Equation 3.5 represents the goal to meet grade requirements in each branch:

$$\sum_{k-l(g)}^{u(g)} y_{bk}(t) + under_{bg}(t) - surplus_{bg}(t) = AU_{bg}(t) \quad for\,all\ \ b,\ g,\ t \qquad (3.5)$$

This equation functions much like Equation 3.4, except it aggregates over subsets of a branch's inventory. The subsets represent officers with the appropriate years of commissioned service to fill authorization targets in a particular branch and grade. For example, for each year, this equation sums the inventory of officers in each branch who have one to four years of commissioned service -- typically lieutenants. Equation 3.5 compares these sums to the lieutenant authorization target for each branch. OAM performs these comparisons in the same fashion for each branch and grade. Again, the auxiliary variables, $under_{bg}(t)$ and $surplus_{bg}(t)$, measure the absolute deviation from attainment of the authorization target for each branch and grade in year *t*.

7. Hard Constraints

Equation 3.6 enforces the policy that mandates the entire accession cohort be distributed in each year:

$$\sum_{b} x_b(t) = ACCOH\ (t) \quad \textit{for each}\ \ t \qquad (3.6)$$

This equation sums over each branch's core accessions, and ensures that this sum equals the accession cohort for year *t*.

Equation 3.7 enforces policy that requires the total officer inventory not exceed a specified amount:

$$\sum_{b} \sum_{k} y_{bk}(t) \leq ITOTAL\ (t) \quad \textit{for each}\ \ t \qquad (3.7)$$

This equation sums the entire inventory of officers in each planning year and ensures that a specified total is not exceeded.

Equation 3.8 ages an initial inventory over the planning horizon:

$$y_{bk}(t) = S_{bt}(k\text{-}t)\, I_{0b}(k\text{-}t) \quad \textit{for each}\ \ b,\ t<k \qquad (3.8)$$

As illustrated earlier, this equation uses the conditional probability that officers in their *kth* year of commissioned service, will survive to the end of year *t*. By multiplying this conditional probability with the initial inventory, this equation generates an inventory for each planning year.

Equation 3.9 relates accessions to the inventory; and, ages new inventory created from accessions:

$$y_{bk}(t) = S_{bk}\, x_b(t-k+1) \quad \textit{for each}\ \ b,\ t \geq k \tag{3.9}$$

This equation multiplies the survival rate -- the unconditional probability an officer survives *k* years -- by core accessions. This yields the number of officers remaining in service that entered in year *t-k+1*.

8. Boundary Conditions

Equations 3.10 and 3.11 represent the policy prohibiting accessions to deviate greatly from year to year:

$$Low\ LYAXS\ (b) \leq x_b(t) \leq Up\ LYAXS\ (b) \quad \textit{for}\ t = 1 \tag{3.10}$$

$$Low\ x_b(t-1) \leq x_b(t) \leq Up\ x_b(t-1) \quad \textit{for}\ t > 1 \tag{3.11}$$

Equation 3.10 represents initial boundary conditions for core accessions into branch *b*. Equation 3.11 represents the boundary conditions on accessions over the planning horizon. These boundary conditions do not allow core accessions to deviate above or below specified proportions, *Low* and *Up*, from the previous year's core accessions. They also have the effect of smoothing change over time.

Equation 3.12 represents the minimum and maximum allowable core accessions into branch *b*:

$$min_b(t) \leq x_b(t) \leq max_b(t) \quad \textit{for each}\ b\ t \tag{3.12}$$

Until now, it has gone without stating that zero accessions for a branch in any year is not acceptable. Equation 3.12 is critical in this regard, as it ensures accessions will occur for each branch. The lower bound represents the minimum number of accessions needed to sustain a branch's total officer authorization requirement, and the upper bound represents training capacities for active duty officers entering each branch in a given year.

D. DETERMINING WEIGHTS

"Given a situation with multiple objectives in which there are no clearly defined weightings for the objectives, no cut-and-dried approach can ever be possible (Williams, 1990)." Research shows that determining the "optimal" set of weights for multi-objective decision making is a sensitive subject embroiled in much debate. For example, Saaty's Analytical Hierarchial Process (AHP), a commonly used method of paired comparisons for determining weights, has been at the center of much debate for its inconsistencies with respect to multiattribute decision making and utility theory (Winkler, 1990). Similarly, another popular approach, Srinivasan and Shocker's Composite Criterion methodology, also proves to be contentious (Srinivasan, 1973). These methods are particularly disturbing to decision makers, as they rely on potentially thousands of "forced choice comparisons", "preferences", and "dominance judgements" concerning a set of attributes -- in this case--branches. Though software exists to help manage such feats, the consequences are clear -- even the most decisive leaders hesitate to consider such comparisons.

The position of this study is that the selection of any particular weighting scheme is moot. It cannot be overemphasized that there is no true single optimizing solution for goal programming models such as OAM. Instead, determining a solution is a process of compromise and satisficing. Thus, often the importance of weighting is overestimated to the detriment of important characteristics of these models -- a choice of solutions, rapid response, and flexibility.

In setting weights for the officer accession model, the principle focus was to ensure that the priorities set forth in the Officer Accession policy were met. That is, the weights should influence the posture of accessions so that: 1) total inventory requirements are met; 2) combat arms captain requirements are met, and 3) all other branch and grade requirements are met as best as possible. The first two requirements seem clear. Since these two requirements are the most important, they receive the greatest weights. The challenge is how to use weights to posture accessions to meet "all other branch and grade requirements as best as possible".

The approach taken, was to provide a weighting scheme that emphasized meeting the relative demand each branch has for a particular grade as suggested by the authorization structure itself. Dividing each branch's grade authorization by its total officer authorization, produces the density of a branch's authorization structure. The authorization density for each branch, in a sense, represents the relative demand for a particular grade category. Multiplying the density by one hundred produces an authorization density histogram.

For example, Figure 4 depicts density histograms for Infantry versus Signal Corps. The stacks, shown in Figure 4, represent the percentage of each branch's authorization allocated towards the five grade categories. For example, the density for majors in the Signal Corps is 22%, while the density for majors in Infantry is 15%. In order to assign a weight that expresses the relative demands for a particular type officer, let the authorization density

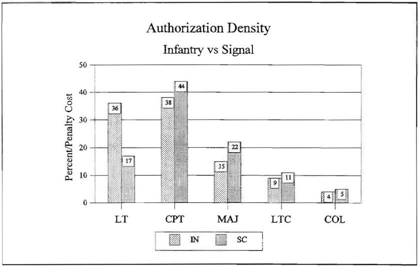

Figure 4. Authorization Density Histograms

represent the penalty cost for a unit of inventory shortage. Thus, from Figure 4, a shortage of one Signal Corps lieutenant incurs a cost of 17, while the penalty cost for an Infantry LT is 36. Conversely, the penalty cost for an IN major is 15, while the penalty cost for as SC major is 22. Seeing that all captain penalty weights for combat arms fell roughly between thirty and forty -- which in general is smaller than the weights for captains in non-combat arms --each penalty weight for combat arm captains was increased to fifty. This was done so that OPMD policy for combat arms captains would be adequately enforced. Summing across the penalties for each branch provided a weight for enforcing the first accession priority -- access to meet the total officer requirements for each branch. Generally speaking, this approach produced weights that fell on a scale between 1 and 115. The penalty weights for combat arms branches are inflated above one hundred because of the increased emphasis for combat arms captains.

Figure 5 depicts the general weighting scheme that resulted by using the branch's authorization density as a measure of the relative demand for a particular type officer. The actual weights do not measure the importance of any particular branch or grade. Instead, they are solely a tool for enforcing the accession policy. Higher weights reflect policy or represent where demands and often choke points are located throughout the Army's branch and grade structure.

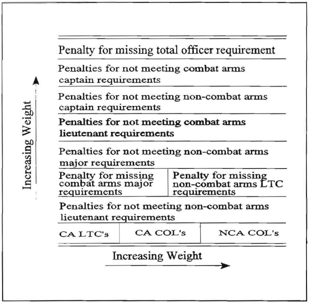
Penalty for missing total officer requirement
Penalties for not meeting combat arms captain requirements
Penalties for not meeting non-combat arms captain requirements
Penalties for not meeting combat arms lieutenant requirements
Penalties for not meeting non-combat arms major requirements
Penalty for missing combat arms major requirements
Penalty for missing non-combat arms LTC requirements
Penalties for not meeting non-combat arms lieutenant requirements
CA LTC's
CA COL's
NCA COL's
Increasing Weight
Increasing Weight

Figure 5. Weighting Scheme

IV. BALANCING LIEUTENANT OVERAGES

The notion of balancing conveys the intent of achieving proportionality. How to formulate a rule that strictly achieves proportionality presents problems that have plagued mathematicians and statesmen for centuries.

> Since the world began there has been but one way to proportioning, namely, by using a common divisor, running the 'remainders' into decimals, by taking fractions above .5, and dropping those below .5; nor can there be any other method. This process is purely arithmetical ... If a hundred men were being torn limb from limb, or a thousand babes were being crushed, this process would have no more feeling in the matter than would an iceberg; because the science of mathematics has no more bowels of mercy than has a cast-iron dog. (Representative John A. Anderson of Kansas, 1882)

In part, the difficulty in achieving proportionality stems from the problem of dealing with fractions. Generally speaking, while strict proportionality is clearly the ideal, ordinarily, it is not met[3]. (Balinski, 1985)

In the context of this study, the problem of proportionality is complicated by other considerations, to include gender issues, the amount of time lieutenants spend in a branch, and limits placed on accessions. Using concepts from stochastic queuing theory, this chapter examines the issue of how to proportion accessions to achieve balance, and presents an extension to OAM to assist OPMD in determining the allocation of officers such that lieutenant overages may balance.

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[3] In OAM, as in many manpower planning models, inventories represent expected values. For convenience, it is common to consider results as approximations, as such, there is no problem with rounding (Vajda, 1978). Although rounding is not generally an issue in manpower planning, in many arenas this is not the case. The reader should reference *The Apportionment of Representation* (Balinski and Young,1985)

A. THE PROBLEM

Control branch accessions are the number of officers to bring into each branch so that lieutenant overages are balanced across a set of receiver and donor branches[4]. Further defined, control branch accessions are core accessions plus or minus branch detailed lieutenants. To distinguish between branches that participate in the Branch Detail Program from those that do not, let us refer to receivers and donors as *players*.

On the surface, balancing overages seems a simple task. In trying to resolve an imbalance, one is inclined to simply transfer excess lieutenants to branches that are out of balance. Although an obvious and perhaps appropriate course of action for a static inventory, "transferring" cannot suffice in a system of lieutenants whose inventory is dynamic. In fact, there are several factors that complicate what might otherwise be a simple task:

1. Training investments and branch specific training requirements, prevent transferring lieutenants that are already in a branch's inventory to solely accommodate the balancing of lieutenant overages.
2. A single year's accessions cannot be used to provide instantaneous balance to the inventory of lieutenants; doing so, would drastically violate limits placed on accessions.
3. Gender constraints limit the extent to which CS and CSS branches can provide the pool of lieutenants needed to achieve balance; and, training capacities limit the extent to which CA branches can accommodate detailed lieutenants.
4. The length of time a lieutenant remains in a branch affects the expected number of lieutenants in that branch.

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[4] Recall that AV, EN, and MP are not included in the set of donor branches as they do not branch detail lieutenants.

Thus, the issue of balancing lieutenant overages is not a matter of shifting inventory. Instead, it is a process of fairly sharing lieutenant overages so that over time, the lieutenant inventory can achieve balance.

The discussion that follows models the lieutenant's years -- the first four years of service -- as a queuing system. Using Little's Law, a well known result from queuing theory, a relationship is derived between the branch detail length and a balanced inventory. The result of this derivation provides direction and insight for determining the fraction of two and four year non-combat arms officers to loan to combat arms branches in order to achieve balance.

An assumption is made that the queuing system is in a state of equilibrium and that its parameters represent steady state averages. Clearly, it is not very plausible that the Army manpower system (or its subsystems) is in *equilibrium* or *steady state*. More appropriately, it is better described as a *transitory system* -- one that moves from one trivial equilibrium to another, on its way to equilibrium (Grinold and Marshall, 1977). Nevertheless, making a steady state assumption adds more to the analysis than it detracts. In fact, making such an assumption is not uncommon in manpower planning:

> The notion of equilibrium is important in the study of physical, social, and economic processes and it plays a central role in our study of manpower flow systems. We do not believe that many manpower systems are in equilibrium. However, the simplifications that result in analyzing an equilibrium system make for a useful approximation to the actual system, and the examination of the equilibrium consequences of any fixed (stationary) policy is essential in uncovering the direction of change implied by the policy and for discovering the policy's long run implications. (Grinold and Marshall, 1977)

Additionally, for the queuing system the attrition that occurs in the lieutenant years is not modeled. Modeling attrition does not add to defining the relationships that will provide direction for determining how to balance the lieutenant inventory. For now, the consequence of removing attrition in this derivation and its subsequent uses is that it underestimates the number of officers to loan from the non-combat arms branches. Accordingly, the results represent lower bounds on the fraction of non-combat arms officers to loan for two and four years. As the reader will see in Chapter V, these estimates of lower bounds provide as much or more insight on the issue of achieving balance than an estimate that considers attrition.

B. THE LIEUTENANT YEARS -- A QUEUING MODEL

We first define (for any queuing system or any subset of a queuing system) the following quantities:

λ = arrival rate -- average number of arrivals entering a system per unit time

L = the average number of persons in the queuing system

W = the average time a person spends in the system

For any queuing system or subset of a queuing system in which a steady state distribution exists, the following is true:

$$L=\lambda W \qquad (4.1)$$

Equation 4.1 is known as Little's Law. This relationship holds regardless of the arrival distribution or queue discipline. (Winston, 1987)

Figure 6 represents the system of lieutenants depicted as a queuing model where R is the set of all receiver branches and D is the set of all donor branches. Each rectangle portrays a queuing subsystem that represents subsets of the lieutenant inventory. Each arrow symbolizes the flow of officers into R and D. λ_R represents the yearly arrival of core accessions into receiver branches -- *the receiver cohort*, and λ_D represents the yearly arrival of core accessions into donor branches -- *the donor cohort*. ϕ and θ represent the fraction of the donor cohort that is loaned to R for four and two years respectively, and $1-\phi-\theta$ is the fraction of non-combat arms officers that are not loaned. L_i represents the average number of lieutenants in a queuing subsystem and W_i represents the average amount of time lieutenants spend in a subsystem.

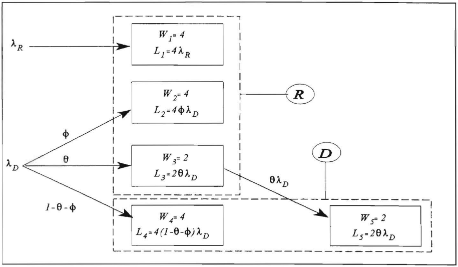
λ_R
$W_1 = 4$
$L_1 = 4\lambda_R$
$W_2 = 4$
$L_2 = 4\phi\lambda_D$
R
ϕ
θ
λ_D
$W_3 = 2$
$L_3 = 2\theta\lambda_D$
$\theta\lambda_D$
D
$1-\theta-\phi$
$W_4 = 4$
$L_4 = 4(1-\theta-\phi)\lambda_D$
$W_5 = 2$
$L_5 = 2\theta\lambda_D$

Figure 6. The Lieutenant Years -- A Queuing Model

Let L_R and L_D represent the expected number of lieutenants in R and D. Thus, from Figure 6, $L_R = L_1 + L_2 + L_3$ and $L_D = L_4 + L_5$. *Let* $AUTH_R$ and $AUTH_D$ represent the sum of the lieutenant authorizations for donor and receiver branches. Attaining proportionality conveys the following intent:

$$\frac{L_R}{AUTH_R} = \frac{L_D}{AUTH_D} \tag{4.2}$$

$$= \frac{L_1 + L_2 + L_3}{AUTH_R} = \frac{L_4 + L_5}{AUTH_D} \tag{4.3}$$

$$= \frac{4\lambda_R + 2\theta\lambda_D + 4\phi\lambda_D}{AUTH_R} = \frac{4\lambda_D - 2\theta\lambda_D - 4\phi\lambda_D}{AUTH_D} \tag{4.4}$$

Solving for θ, we form a relationship between the proportion of two year details (θ) versus four year details (ϕ) -- Equation 4.5. For compactness, $AUTH_R$ and $AUTH_D$ are abbreviated with A_R and A_D.

$$\theta = -2\phi + \frac{2}{\lambda_D}\,\frac{(\lambda_D A_R - \lambda_R A_D)}{(A_R + A_D)} \tag{4.5}$$

Equation 4.5 has the form of the line, $\theta = m\phi + b$, where $0 \leq \theta \leq 1$, and $0 \leq \phi \leq 1$. Thus, given a two and four year detail program, a set of authorizations, and the yearly accessions into donor and receiver branches, the equation of the line with slope $m = -2$, and intercept b (shown in Equation 4.5), represent trade-offs between the fraction of officers to loan for two versus four years. The points along that line represent the set of lower bounds on the proportion of four and two year detail officers necessary to attain a balanced inventory. In Chapter V these equations assist in providing insight on the goal of balancing lieutenant overages. Meanwhile, we examine the usefulness of these relationships, in preparation for their incorporation into the officer accession model.

Let $\hat{\alpha}$ be defined as the proportion of the total inventory of players, with respect to the total lieutenant authorizations for players -- Equation 4.6.

$$\hat{\alpha} = \frac{L_R + L_D}{AUTH_R + AUTH_D} \tag{4.6}$$

In Equation 4.6, the numerator represents the expected number of player lieutenants in the inventory given no attrition. Given no attrition, to achieve balance among the set of player branches, the goal would be that each branch attain the ratio, $\hat{\alpha}$.

Equation 4.2 represents a balance in the inventory of R and D with respect to the authorizations in R and D. Balancing each player's lieutenant inventory would require that each $r \in R$ and each $d \in D$, attain the proportion $\hat{\alpha}$ with respect to their lieutenant authorizations. Equations 4.2 and 4.4 suggest that in order to achieve symmetry across all players, OPMD should balance -- with respect to a branch's lieutenant authorization -- the proportion of core, two, and four year details entering or leaving each branch. Using these insights and relationships, the next section presents an extension to OAM, called the Officer Accession/Branch Detail Model (OA/BDM).

C. OFFICER ACCESSION/BRANCH DETAIL MODEL (OA/BDM)

The Officer Accession/Branch Detail Model determines both core and control branch accessions for each year in a planning horizon. The model extends OAM by incorporating OPMD policy that seeks to balance the lieutenant inventory. In sum, additions to OAM include three goal constraints, three hard constraints, four variables, and two additional boundary conditions. The model also requires several new data inputs.

The Officer Accession/Branch Detail Model (OA/BDM) extends the composite objective function of OAM by adding auxiliary variables that represent the absolute positive and negative deviations from a target proportion $\alpha(t)$. The set of goal constraints are extended by including goals that harness the relationships derived in the previous section. Subject to constraints imposed by gender issues, training capacities, and boundary conditions, the goal constraints strive to balance projected lieutenant inventories for player branches by solving for the number of two and four year detail officers to loan to receiver branches. The set of hard constraints are extended to include: a) a relationship that determines for each year the target proportion $\alpha(t)$ to be attained by each player; b) a relationship that limits the donor population, based upon projected amounts of males and females in each donor branch; and c) an expression that limits accessions into receiver branches, based upon training capacities. Boundary conditions are extended to include limits on changes in the number of detailed officers from year to year.

The full model is implemented using the General Algebraic Modeling System. The computer code for the full model is located in the Appendix A. The following algebraic formulation addresses only those aspects of OA/BDM not originally contained in OAM.

1. Indices

This model extends the subsets of OAM by redefining subsets of b, to become:

r	Receiver branches	*IN, AR, FA, AD, CM*
d	Donor branches	*SC, FI, TC, OD, QM, MI, AG*
$d2$	Two year donors	*SC, FI, TC, OD, QM*
$d4$	Four year donors	*MI, AG*
n	Non-detail branches	*AV, EN, MP*

2. Data

$LYDET(r,d)$	Previous year's details into r from d
ρ	Max proportion of male officers to detail from donor branches
γ_d	Fraction of female officers in branch d
$S(k)$	Army average that officers will survive their *kth* year of service
Δ^+, Δ^-	Max and min proportions of change for branch detailed officers

3. Variable Definitions

a. *Decision Variables*

$det_{rd}(t)$	The number of officers to loan to r from d in year t
$\alpha(t)$	The target level of balance in year t
$obal_b(t)$	Fraction of overbalance in lieutenants in branch b in year t
$ubal_b(t)$	Fraction of underbalance of lieutenants in branch b in year t

4. The Objective Function

$$Min \sum_{t}^{T} \delta(t) \left[\begin{array}{l} \sum_{b} bpen_{b}(t)\ bunder_{b}(t) + \sum_{b} \sum_{g} w_{bg} under_{bg}(t) \\ + \sum_{b} obal_{b}(t) + ubal_{b}(t) \end{array} \right] \tag{4.7}$$

Equation 4.7 represents the new objective function for OA/BDM. In addition to the objectives described in Chapter III, OA/BDM enforces OPMD policy to balance lieutenant overages. By minimizing both the positive and negative deviational variables associated with a target proportion, *α(t)*, OA/BDM determines control branch accessions that balance, as best as possible, projected inventories among player branches.

5. Goal Constraints

Equation 4.8 represents the goal to achieve balance among lieutenants for receiver branches:

$$\frac{\sum_{k=1}^{4} x_{r}(t) s_{rk} + \sum_{k=1}^{2} \sum_{d2} det_{r,d2}(t) s_{rk} + \sum_{k=1}^{4} \sum_{d4} det_{r,d4}(t) s_{rk}}{AU_{r,LT}(t)} + ubal_{r}(t) - obal_{r}(t) = \alpha(t), \forall r, t \tag{4.8}$$

Eq 4.8 determines the core and detail officer accessions, into each receiver branch such that the projected proportion of lieutenant inventory to lieutenant authorization is balanced, as best as possible, across the set of receiver branches. Examining the numerator, the first summation represents (in steady state) the expected number of core accessions remaining in a receiver branch. Likewise, the next two sets of summations represent the expected number of two and four year detail officers in a receiver branch. Thus the numerator is the expected number of lieutenants (assuming steady state) in a receiver branch. The ratio -- the projected number of lieutenants in a receiver branch, over its authorization for lieutenants -- is compared to the target proportion $\alpha(t)$. The auxiliary variables $obal_r(t)$ and $ubal_r(t)$ measure the absolute deviation from the target proportion. They represent the fraction over and under the target level of balance.

Similarly, Equations 4.9 and 4.10 represent the goal to achieve balance among lieutenants for two and four year donors, respectively:

$$\frac{\sum_{k=1}^{4} (x_{d2}(t) - \sum_{r} det_{r,d2}(t))s_{d2,k} + \sum_{k=3}^{4} \sum_{r} det_{r,d2}(t) s_{d2,k}}{AU_{d2,\,LT}(t)} + ubal_{d2}(t) - obal_{d2}(t) = \alpha(t), \forall\ d2, t \quad (4.9)$$

$$\frac{\sum_{k=1}^{4} (x_{d4}(t) - \sum_{r} det_{r,d4}(t))s_{d4,k}}{AU_{d4,LT}(t)} + ubal_{d4}(t) - obal_{d4}(t) = \alpha(t), \forall\ d4, t \quad (4.10)$$

Focusing on the numerators, the first summation term in Equation 4.9, and the complete numerator of Equation 4.10, represent the expected number of officers that are not loaned, remaining in a donor branch. In Equation 4.10, the second term in the numerator represents the expected number of returning detail officers that are in the two year donor branch.

6. Hard Constraints

Equation 4.11 represents the proportion of the projected total inventory of lieutenants to the total authorization for lieutenants in the player branches:

$$\alpha(t) = \frac{\sum_{k=1}^{4} S(k) \sum_{b \in n} x_b(t)}{\sum_{b \in n} AU_{b,LT}(t)}, \quad \forall\, t \qquad (4.11)$$

The parameter *S*(*k*) is the Army average survival rate that an officer will survive *k* years of commissioned service. To achieve balance among the set of player branches, the goal would be that each branch attain the ratio, $\alpha(t)$. Thus, $\alpha(t)$ represents the target proportion for all branches.

Equation 4.12 constrains control branch accessions for receiver branches:

$$x_r(t) + \sum_{d} det_{r,d}(t) \leq max_r(t) \quad \forall\ r,t \qquad (4.12)$$

This is done by adding core accessions to the number of branch detailed officers entering a particular branch, and ensuring that this sum does not exceed established training capacities.

Equation 4.13 establishes an upper bound on the amount of officers to loan from donor branches:

$$\rho\ (1-\gamma_d)\ x_r(t) - \sum_r det_{r,d}(t) \geq 0 \quad \forall\ d, t \qquad (4.13)$$

Recall that γ_d is the fraction of accessions into d that are female. Thus , $1-\gamma_d$ multiplied by accessions is the number of male officers in the a donor's cohort. Recall also that ρ is the fraction of male officers that can be detailed -- a parameter supplied by the decision maker. Equation 4.13 allows the decision maker to control how much of the "detailable" population to donate to receiver branches.

7. Boundary Conditions

Equations 4.14 and 4.15 allow the decision maker to place limits on the amount of change in the number of branch detailed officers between years:

$$\Delta^- LYDET\ (r,d) \leq det_{r,d}(t) \leq \Delta^+ LYDET\ (r,d) \quad for\ t=1 \qquad (4.14)$$

$$\Delta^- det_{r,d}(t-1) \leq det_{r,d}(t) \leq \Delta^+ det_{r,d}(t-1) \quad for\ t>1 \qquad (4.15)$$

As before, these boundary conditions have the effect of smoothing change over time.

V. MODEL DEMONSTRATION & RESULTS

The principal effort of this study has been to develop a model that assists the Officer Personnel Management Directorate in determining officer accessions and to equip them with an analysis tool to evaluate the impact of planned accessions. The modeling efforts put forth resulted in the development of the Officer Accession Branch Detail Model (OA/BDM). This chapter is meant to demonstrate how OPMD can use OA/BDM to evaluate planned accessions. Also presented is an analysis of the current two and four year detail program using the relationships derived from the queuing model of Chapter IV.

A. MODEL DEMONSTRATION

To demonstrate OA/BDM and to gain a measure of its effectiveness as an analysis tool, this study uses OA/BDM to assess the impact of recommendations and assumptions made by the steady state officer accession model currently employed by the Distribution Division of OPMD. To do this, OA/BDM is used to create a steady state environment and to implement accession solutions produced by OPMD's steady state officer accession model. The accession feedback from OPMD's accession plan serves as a basis for comparison with results generated by OA/BDM's accession plans. The demonstration focuses on two questions: 1) How well do steady state accession plans meet OPMD accession objectives? and 2) Are average officer survival rates an adequate predictor of the Army's ability to retain officers in each career branch?

B. DEMONSTRATION PROTOCOL

To examine these questions, this study uses OA/BDM to conduct two tests. Each test consists of two runs with OA/BDM -- a baseline run which implements a predetermined accession plan from OPMD's steady state model, and a second run which allows OA/BDM to recommend its own accession plan.

1. Demonstration Test I

This test evaluates accession plans generated by OA/BDM and the OPMD officer accession model while employing Army average survival rates. During *Test I,* OA/BDM implements the same input parameters used by the OPMD officer accession model to determine FY '95 accessions. These parameters are established as constant over a specified planning horizon, thus creating a steady state environment. Two model runs are executed.

In the first run, decision variables are fixed at levels that represent core and control branch accessions recommended by OPMD's steady state model for FY '95. Using the FY '88 - '89 Army average survival rates employed to determine FY '95 accessions, OA/BDM ages initial and subsequent inventories that result from the OPMD officer accession plan. We refer to a test run under these conditions as *Baseline I.*

In the second run, decision variables are set free. This allows OA/BDM to recommend its own accession plan. OA/BDM ages the same initial inventory as the first run, but determines accessions based on the available inventory to fill the FY '95 authorization targets. We refer to this run as *OA/BDM I.* The inventory feedback from both model runs are used to examine how well the accession plans satisfy OPMD's officer accession policy.

2. Demonstration Test II

This test uses OA/BDM to evaluate whether the Army average survival estimators are good predictors of officer retention within each branch. With the exception of survival rates, *Test II* implements the same input parameters used by the OPMD officer accession model in determining FY '95 accessions. These parameters are established as constant over a specified planning horizon. Two model runs are executed. During these runs, OA/BDM uses FY '88 - '89 branch specific survival rates to age initial and subsequent accessions produced by itself and by the OPMD accession model.

In the first run, decision variables are fixed at levels that represent core and control branch accessions recommended by the steady state model for FY '95. These fixed accessions represent decisions made using FY '88 - '89 Army average survival rates; however, using the FY '88 - '89 branch specific survival rates, OA/BDM ages the initial inventory, and the subsequent inventory generated from the OPMD officer accession plan. We refer to a test run under these conditions as *BaselineII*.

In the second run, OA/BDM is executed with the decision variables set free. As before, OA/BDM is then used to age the same initial inventory but to determine accessions based on the available inventory to fill the FY '95 authorization targets. We refer to this run as *OA/BDM II*.

If the FY '88 - '89 Army average survival rates are good predictors of survival in all branches, then one expects that resulting inventories from the two model runs would be close. Thus the feedback from both model runs can be used to determine whether Army average survival rates are empirically good predictors of officer retention in each branch. Furthermore, we can assess the potential impact that these estimates have on OPMD's ability to meet officer accession objectives.

C. MEASURES

In order to provide a means for comparison, several measures of effectiveness (MOE's) are examined. First, the values minimized by OA/BDM's objective function -- the total penalty cost -- provide quantitative measures for comparing the accession schedules produced by the two models. Recall that the weighted composite objective function of OA/BDM minimizes shortfalls between desired inventory levels and authorization targets, and that the model applies penalties for each shortfall. Accordingly, objective function values represent the total penalty costs that result from missing desired targets. Therefore, the smaller the penalty cost, the closer an accession plan meets OPMD's officer accession policy.

Second, three OPMD policy objectives were selected to provide quantitative and qualitative measures of the results produced by both models. By qualitative we mean, "How well do scheduled accessions satisfy OPMD policy?" The accession MOE's chosen were: a) How well do accessions satisfy the total officer authorization requirements in each branch? b) How well do accessions meet the need for combat arms captains? and c) How well do accessions balance projected lieutenant inventories?

D. DEMONSTRATION RESULTS

OA/BDM was implemented using the General Algebraic Modeling System (GAMS ver. 2.25) on a 486DX2 PC, using the XA Solver from Sunset SoftwareTechnology (1994). A fifteen year planning horizon (1996 -2010) was chosen so that the accession MOE's might be adequately measured. The baseline runs generated 10,666 variables and 8,326 constraints. The execution time for a baseline run was approximately four minutes. Runs of OA/BDM generated 10,666 variables, and 10,006 constraints. Each run took approximately 24 minutes.

The output from these models is an inventory of officers by years of commissioned service for each period in the planning horizon. Although OA/BDM produces eleven reports that present the detailed inventories for further analysis, the extensive nature of the output precludes its reproduction here. Consequently, the results that follow summarize the outcomes of the two demonstrations.

1. Results: Demonstration Test I

To provide a quantitative measure of how well each accession schedule met all accession objectives, the penalty costs were examined. The total penalty costs assessed upon OPMD's steady state accession plan in *Baseline I* was 835,562. The total penalty costs assigned to *OA/BDM I* was 784,438. This suggests that *OA/BDM I's* accession plan better fulfills the accession objectives.

With regard to meeting total officer authorizations, only Chemical Corps was identified as having a shortage by both models. *Baseline I* did not resolve the shortage of chemical officers until the year 2000 -- the fifth planning year. *OA/BDM I* resolved the shortage by the year 1997 -- the second planning year. This is consistent with the steady state model employed by OPMD. As stated earlier, the OPMD accession model does not consider the current inventory to determine accessions.

Figure 7 depicts the impact of each models' planned accessions on the goal to meet combat arms captain requirements. With only slight deviations over the planning horizon, both accession plans performed similarly. Notice that the first three years reflect a decline in meeting captain requirements. This is not caused by either model, but is a reflection of the initial inventory used in the model. The impact of accessions on captain authorizations cannot be observed until the initial inventory of lieutenants is aged through the year 2000.

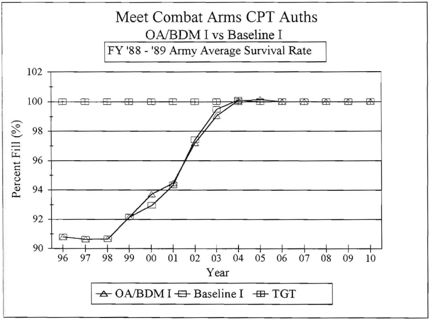

Figure 7. Percent Fill of Combat Arms Captains: OA/BDM I vs Baseline I

Next, the OPMD goal to balance player branch lieutenant inventories is evaluated. Recall that OA/BDM attempts to level projected inventories of officers in each of the twelve player branches by determining a target proportion that should be attained in order for player branches to achieve balance. Figure 8 depicts the number of branches in each year that met the target level of proportion. Results indicate the steady state model achieved balance among three of the player branches while *OA/BDM I* achieved balance among eight of the twelve player branches. Figure 8 also suggests that eight balanced branches was the best that could occur given the constraints imposed on the model. The specifics of this problem are discussed later in this chapter. Meanwhile, Table 5-1 summarizes the results of Test I.

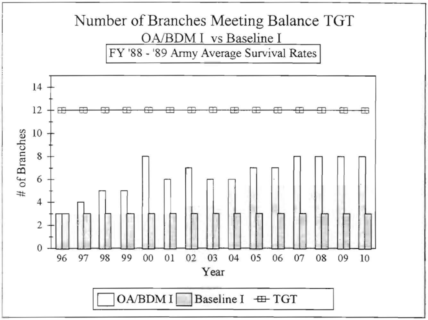

Figure 8. Balance Lieutenant Inventory: OA/BDM I vs Baseline I

Summary of Results: Demonstration Test I

MOE	Baseline I	OA/BDM I
Penalty Cost	835,562	784,438
Meet Total Officer Auths	CM short officers through the year 2000	CM short officers through the year 1997
Meet CA CPT Auths	All CA CPT requirements met by the year 2005 and sustained through the year 2010	All CA CPT requirements met by the year 2004 and sustained through the year 2010
Balance LT Inventory	25% of player branches attained target proportion over the 15 year horizon	25% of player branches attained target proportion in the first planning year, 67% by year 2000

Table 5-1. Baseline I vs OA/BDM I using Army average survival rates for FY '88 - '89

2. Results: Demonstration Test II

Recall that *Test II* was conducted to assess the impact of the -- FY '88-'89 Army average survival rates on OPMD's ability to attain officer accession objectives. Thus, another model run was conducted -- *Baseline II* which implemented FY '95 steady state accessions into an environment in which branch specific survival rates prevailed. The stocks of manpower generated by *Baseline II* represent inventories of officers that were aged using branch specific survival rates, but whose accessions were determined using a single estimate of survival. This was compared to another model run -- *OA/BDM II* -- with the model free to determine accessions.

Again, total penalty costs between *Baseline II* and *OA/BDM II* were compared. The total penalty costs levied against OPMD's steady state accession plan in *Baseline II* was 850,081. The total penalty costs assigned to *OA/BDM II* was 780,731. This represents a significant decrease in the ability of the steady state accession plan to accommodate OPMD policy. Conversely, *OA/BDM II' s* accession plan represents an increase in its ability to meet all OPMD policy objectives. In fact, total penalty costs from *Baseline II* exceeded penalties assessed during *Test I*, while *OA/BDM II's* penalty costs signify improvement over *Test I*. Thus, *OA/BDM II's* results suggest that there is a discernable value in the information provided by the branch specific retention rates that resulted in a "savings" of 69,350 penalty points over the planning horizon.

With regard to meeting total officer authorizations the steady state model allowed a shortfall in Signal Corps officers in 1996, as well as a shortfall in Chemical officers through the year 2000. *OA/BDM II's* performance did not change from *Test I* -- there was a shortage in Chemical Corps officers through the year 1997.

With regard to meeting the combat arms officer requirements, Figure 9 depicts that the accession plan that the steady state model provided in *Baseline II* did not meet this objective during the entire planning horizon; *OABDM II* met and exceeded total requirements by two percent. Beginning in the year 2000, *OABDM II* began a rapid improvement over the

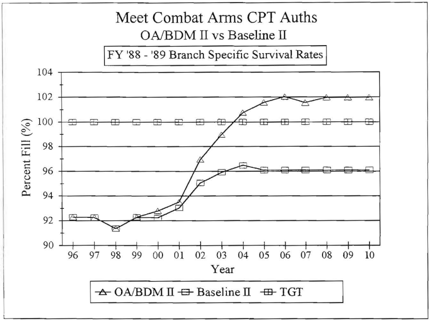

Figure 9. Percent Fill of Combat Arms Captains: OA/BDM II vs Baseline II

steady state accession plan and ultimately exceeded objectives. Meanwhile, the best that the steady state model could attain was a 96% level of fill in combat arms captain requirements.

With regard to balancing lieutenant inventories, Figure 10 represents that the steady state model allowed two branches to attain target proportions while *OA/BDM II* was again limited to eight player branches attaining target proportions. This was however an improvement over *OA/BDM I*. *OA/BDM II* was able to use additional information about the branches to achieve target levels seven times during the planning horizon, whereas *OA/BDM I* was successful only five times during the planning horizon. Table 5-2 summarizes the results of Test II.

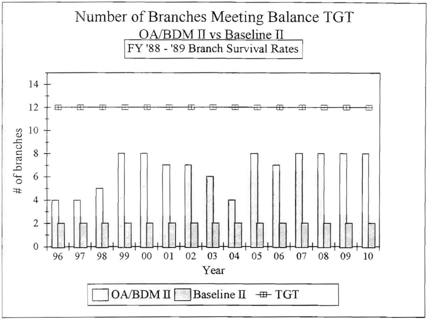

Figure 10. Balance Lieutenant Overages: OA/BDM II vs Baseline II

Summary of Results: Demonstration TEST II

MOE	Baseline II	OA/BDM II
Penalty Cost	850,081	780,731
Meet Total Officer Auths	SC short officers in 1996; CM short officers through year 2000	CM short officers through year 1997
Meet CA CPT Auths	By year 2002, meets and sustains only IN CPT requirements; no other CA branches meet their Auth in the 15 year planning horizon	Three CA branches meet Auths. by 2003; All CA CPT requirements met by year 2006 and sustained through the year 2010
Balance LT Inventory	16% of player branches attain target balance over the 15 year horizon	25% of players branches attain target balance in first planning year, 67% by year 2000

Table 5-2. OA/BDM II vs Baseline II using FY '88 - '89 branch specific survival rates.

In summary, the results indicate that accession recommendations produced by OA/BDM provide a schedule of accessions that improve OPMD's ability to meet officer accession objectives. As reflected in Table 5-2, the results also indicate that the FY '88-'89 Army average survival rates overestimate the Army's ability to retain combat arms captains. It is important to note that the purpose of this exercise was to demonstrate the ability of OA/BDM to be used as a tool for analyzing an accession plan. Contrasts between OA/BDM and the OPMD officer accession model are not surprising and can be attributed to the principal difference between these two models: the baseline model uses a steady state approach to determining core accessions whereas OA/BDM implements a dynamic approach.

With respect to balancing lieutenant overages, OA/BDM was only able to achieve balance among lieutenant inventories in 66% of the player branches. Although not shown here, results indicated that IN, AG, FI, and QM, were consistently out of balance with the other player branches. AG, FI, and QM, were consistently overbalanced, while IN was consistently underbalanced. This imbalance indicates, that the donor population could not loan enough lieutenants to provide balance across the set of all receiver branches. This is consistent with what we know about the "detailable" populations in these donor branches but perhaps there are other forces that limit player branches from attaining balance. In the next section this study seeks to provide insight on the following questions:

1) How can the Army achieve balance among player branches?

2) What is the proportion of the lieutenant population that should be loaned to the combat arms in order to achieve balance?

3) What is the impact of detail length on balancing lieutenant overages?

E. ANALYSIS OF THE CURRENT BRANCH DETAIL PROGRAM

The queuing model developed in Chapter IV can be used to assess the viability of the current two and four year detail program in achieving the objective of balancing lieutenant overages. Equation 5.1 represents the relationship between the proportion of two year versus four year details:

$$\theta = -2\phi + \frac{2}{\lambda_D}\frac{(\lambda_D A_R - \lambda_R A_D)}{(A_R + A_D)} \tag{5.1}$$

where θ represents the proportion of two year details, and ϕ represents the proportion of four year details. Thus, given a set of authorizations, and the receiver and donor cohorts, one can use this relationship to assess the stationary impact of a particular year's accessions on the notion of balancing lieutenant overages. For example, substituting the values for the FY '95 core accession plan into Equation 5.1, yields the following relationship shown in Figure 11:

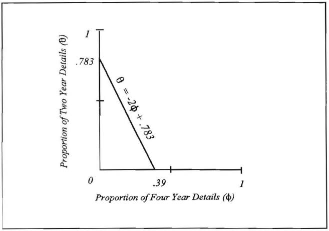

Figure 11. FY '95--Two versus Four Year Detail

The line in Figure 11 represents the trade-offs between the two and four year donor proportions necessary to attain balance. One can use this relationship as a model for assessing the branch detail policy. For example, if there were no two year details in 1995, then the model suggests that OPMD donate 39% of the donor cohort for four years. Likewise, if there were no four year details, this model suggests that OPMD donate 78% of the donor cohort for two years. In FY '95, OPMD detailed 40% of the donor cohort. Fifteen percent of the donor cohort were four year details, and the remaining 25% were two year details. Thus, this model suggests it would have been necessary to loan the entire FY '95 detail population for four years in order to attain balance between player branches. This model also suggests that if 15% of the donor cohort is the desired target level for four year details, then OPMD would need to loan 48% percent of the donor cohort for two years. This result implies that OPMD would need to loan 63% of the donor cohort in order to achieve balance -- an amount not feasible given boundary conditions on core and control branch accessions.

In summary, using the FY '95 accession data revealed that the current two and four year detail program is not suited to the objective of balancing the lieutenant inventory. As a means for assessing alternate branch detail courses of action, similar relationships can be derived that represent various combinations of detail lengths or standard detail lengths. For example, a similar analysis using FY '95 data was conducted using a standard length of three years. Results indicated that, a standard detail length of three years, required loaning 52% of the donor cohort to receiver branches.

VI. SUMMARY & CONCLUSIONS

The purpose of this study was to develop a flexible, responsive manpower optimization model to assist Army personnel planners in determining the yearly allocation of officer accessions, and to provide a model that allows the Army's Officer Personnel Management Directorate to investigate the long term implications of planned accessions. The modeling efforts put forth in this study resulted in the development of the Officer Accession Branch Detail Model (OA/BDM). This thesis also investigated the Army Branch Detail Program and its impact on the problem of balancing lieutenant overages. Using queuing theory, this study derived and modeled relationships that suggest the appropriate numbers of officers to loan from donor to receiver branches in order to attain balance. This chapter concludes the study by presenting uses and capabilities of OA/BDM, model assumptions and limitations, areas for future study, and finally, conclusions.

A. MODEL APPLICATIONS AND CAPABILITIES

In Chapter V, this study demonstrated how OA/BDM can be used to examine planned officer accessions by setting accession variables at a constant level. It is possible however to implement into OA/BDM any program of accessions to measure the program's impact on meeting authorization targets. Conversely, it is possible to change the authorization parameters of the model to evaluate impacts on resulting accessions. Additionally, because the model outputs represent inventories, the Army's projected inventories can be studied to suggest changes to branch structures.

In the formulation presented in Chapter III, OA/BDM is constrained so that the accession cohort from DCSPER is exactly satisfied. This was done in accordance with the current accession policy to distribute the entire cohort. However, by relaxing this constraint, OA/BDM can be used to determine only the necessary number of accessions needed in each branch. The implication is that the model can be modified from an allocation model, to a tool that economizes and considers the actual costs of its accession decisions. Furthermore, by completely suppressing the cohort constraint, OA/BDM has a role in recommending the actual number of accessions needed to sustain a particular schedule of authorizations over a planning horizon.

B. MODEL LIMITATIONS AND ASSUMPTIONS

The Officer Accession Branch Detail Model serves many different purposes; however, it is not without limitation. It is important to understand that the inventory forecasts of the model should "...never be interpreted as what *will* happen but what *would* happen if the assumed trends continue (Bartholomew, 1979)." OA/BDM relies heavily on survival input data, as such, model results are purely deterministic. Also, not considered in the model are the effects of "below-the-zone" promotions and the effects of "promotable inventory" on meeting authorization requirements. Because the model aggregates the inventory by years of commissioned service that "typically" represent a particular grade category, the model becomes sensitive to parameters that define the upper and lower limits on years of commissioned service for a particular grade.

Finally, this study assumes that branch detail officers survive the same as all other officers. If this assumption is valid, then the study assertion that loaning lieutenants does not impact the Army's ability to meet requirements beyond the grade of lieutenant would be true. However, if it can be shown that a branch detail officer has a particularly strong or poor survival rate, i.e. a unique distribution for survival, then the study assertion would be false and the need arises to model this behavior.

C. AREAS FOR FUTURE STUDY

This study recommends two areas related to the *Officer Accession /Branch Detail* problem and OA/BDM for future research: 1) an investigation of the survival distribution of officers that are loaned versus officers that are not loaned; and 2) studies related to evaluating end effects for multi-year planning models such as OA/BDM.

Studies to test the assumption that branch detail officers survive as well as all other officers should be done as a preliminary step for determining if OA/BDM should be enhanced to account for differences between officers that are loaned and officers that are not loaned. This study demonstrates that branches behave differently, and that average survival rates might overestimate our ability to retain certain officers. This study also demonstrates that models such as OA/BDM can use this information to posture accessions so that future demands can be met. If there is a propensity for branch detail officers to either survive or attrite at higher rates than other officers, then this can affect the Army's ability to meet requirements beyond the grade of lieutenant. If this situation exists, model enhancement is warranted.

End effects refers to a problem that surfaces in multiyear planning models. The problem of end effects arises any time one uses a finite planning horizon to model what is essentially an infinite or indeterminant time horizon. For example, in the context of this study, it would be common practice to execute a fifteen or twenty year planning horizon, rather than a thirty year planning horizon to get feedback on short range plans concerning accessions. The length of the planning horizon is usually subjective and often determined by the nature of the problem, and knowledge of the data and the model. It is known however that imposing artificial finite horizons can affect the optimal solution (Walker, 1995). Of interest to this study is to gauge, control, and understand the impact of end effects on any particular problem instance employing OA/BDM. A study by Walker (1995) showed that primal and dual approximation methods can be successfully employed to identify and quantify end effects in finite manpower planning models. Applications of these methods may be of value to potential users of OA/BDM.

D. CONCLUSIONS

In conclusion, this study demonstrates that the application of multiyear weighted goal programs such as OA/BDM provides OPMD the capability of rapidly producing and or evaluating planned accessions. Unlike many manpower planning models that rely heavily on steady state assumptions, OA/BDM provides a dynamic multiyear approach for determining officer accessions. Furthermore, the model's flexibility suggests that it can be used for a host of different purposes other than officer accessions.

With regard to the goal of balancing lieutenant overages, this study found that the current two and four year detail program does not provide a viable means for balancing the lieutenant inventory. The analysis conducted using FY '95 data suggests that standard three or four year programs, or a combination thereof, offer a more tractable means for achieving balance among player branches.

APPENDIX A. OA/BDM GAMS CODE

```
$TITLE  OFFICER ACCESSION/BRANCH DETAIL MODEL  (OA/BDM )
$STITLE JEFFREY CORBETT, SEPT 5, 1995

*------------GAMS AND DOLLAR CONTROL OPTIONS-------------------------

$OFFUPPER OFFSYMLIST OFFSYMXREF

OPTIONS
  LIMCOL = 0   , LIMROW  = 0    ,  SOLPRINT = OFF , DECIMALS = 2,
  RESLIM = 5000, ITERLIM = 1000000, OPTCR   = 0.1 , SEED     = 3141,
  EJECT,        LP=XA ;
*-----------------------------------------------------------------------------------------
$ontext
                                    OA/BDM

 This model determines core and control branch accessions such that OPMD
 officer accession objectives are met. OA/BDM ages the current inventory over
 a specified planning horizon and determines core accessions for each year
 based upon the available inventory to fill a set of authorization targets. The model
 also determines control branch accessions such that projected lieutenant inventories
 balance as best as possible.

 OABDM uses survival/retention rates for each  branch to compute the conditional
 probability that officers in their kth year of commissioned service will survive
 through year t. By implementing branch specific survival rates, this model assumes
 that individuals in each branch survive at different rates.

 Comments:
 Years of Commissioned Service = Current Year - Accession Year + 1
 example: John Doe was accessed in 1985, and in 1995 he is in his 11th YOCS

 Inputs
 -Initial inventory of officers by branch by years of commissioned service
 -Accession cohort from DCSPER
 -Previous year's core accessions
 -Previous year's detail accessions
 -Survival rates (branch specific survival rates)
 -Authorization data
 -Total authorization by branch
 -Penalty weights for missing a grade in branch b
 -Discount factor
 -Bounds on core accessions
 -Proportion of female officers in donor branches
 -Proportion of detailable population to donate
 -Total officer inventory limit
 -Bounds on the proportion of change between time periods

  Processes
```

```
*Age the inventory
*Determine accessions using such that:
 -Total branch inventory requirements are met
 -The need for combat arms captain are met
 -Shortfall from authorizations in branch and grade are minimized
 -Balance projected lieutenant inventories
 -The entire cohort is distributed
 -The total Army Competitive Category officer limit is not exceeded
 -Change is limited from period to period


 Outputs
 -Core Accessions by year
 -Control Branch Accessions by year
 -Percent change in core accessions from year to year
 -Officer Strength report
 -Percent Fill Report
 -Core branch overages and underages by year
 -Total inventory report by year
 -Detail report
 -Inventory Balance report
 -Summary report

$offtext


*---------------------------INDICES-----------------------------------

SETS
 B      branches              /IN,AR,FA,AD,CM,SC,FI,TC,OD,QM,MI,AG,
                              AV,EN,MP/
 R(B)   receiver branches     /IN,AR,FA,AD,CM/
 D(B)   donor branches        /SC,FI,TC,OD,QM,MI,AG/
 D2(D)  two year donors       /SC,FI,TC,OD,QM/
 D4(D)  four year donors      /MI,AG/
 N(B)   non detail branches   /AV,EN,MP/
 G      grades                /LT,CPT,MAJ,LTC,COL/
 K      years of service      /1*30/
 T      planning horizon      /1996*2010/
 L(K)   lieutenant's years    /1*4/
 C(K)   captain's years       /5*11/
 M(K)   major's years         /12*16/
 U(K)   lt. colonel's years   /17*22/
 O(K)   colonel's years       /23*30/
 J      bounds                /LOWER, UPPER/;

*------------------------------DATA-------------------------------------
SCALAR

 UP          maximum proportion of increase between time periods /*.*/
 LOW         maximun proportion of decrease between time periods /*.*/
 ITOTAL      limit on total officer inventory                    /*****/
 RATE           foresast discount rate                                  /*.*/
 RHO            max fraction of males that can be detailed              /*.*/
 PDELTA         max proportion of increase in details between years    /*.*/
 NDELTA         max proportion of decrease in details between years    /*.*/

TABLE
$INCLUDE .\AUTHS.97
TABLE
$INCLUDE .\TOTAL15.TXT
TABLE
$INCLUDE .\NVENTORY.95
TABLE
$INCLUDE .\LIMIT.TXT
TABLE
$INCLUDE .\PENALTY.TXT
TABLE
$INCLUDE .\RETAIN.TXT

PARAMETERS
$INCLUDE .\COHORT15.TXT
$INCLUDE .\LYACCESS.TXT
$INCLUDE .\LYDETAXS.TXT
$INCLUDE .\FEMLIMIT.TXT
$INCLUDE .\RETAIN1.RTS


PARAMETER
 DISCOUNT(T)   present discounted value of a forecast in year t;
 DISCOUNT(T) = (1/(1+RATE))**(ORD(T)-1);
*DISPLAY DISCOUNT;

PARAMETER
 BPEN(B)           weight for missing total officer auth in branch b;
 BPEN(B) = SUM(G,WU(B,G));
*DISPLAY BPEN;

PARAMETERS
 S(B,T,K)             rate that officers in their Kth YCS survive to end of year T;
 S(B,T,K)$(( ORD(K)+ORD(T)) LE CARD(K)) =   RETAIN(B,K+ORD(T))/RETAIN(B,K);
*DISPLAY S;

*---------------------------- VARIABLES-------------------------------
VARIABLES
 V                 minimize deviation from goals

*-----------------------------DECISION VARIABLES---------------------
POSITIVE VARIABLES

 X(B,T) accession to branch B in time period T
 DET(R,D,T)      officers to loan to R from D
 ALPHA(T)        target proportion of projected LT inventory for each branch
```

```
*-----------------------------INVENTORY VARIABLES---------------------------

 Y(B,K,T)        inventory of officers in branch B in their Kth YCS in time T

*-----------------------------AUXILLIARY VARIABLES---------------------------

 UNDER(B,G,T)         shortfall for branch B in grade G in time T
 SURP(B,G,T)          surplus for branch grade G in time T

 BSHORT(B,T)          shortfall from total branch authorization in time T
 BOVER(B,T)           number of officers over the branch authorization in time T

 OBAL(B,T)            fraction of lts overbalanced in B in time T
 UBAL(B,T)            fraction of lts underbalanced in B in time T

*--------------------------EQUATION DEFINITIONS-----------------------------

EQUATIONS

 OBJ                  the objective function

 LCBT(R,G,T)          meet auth goals for LTS in combat arms
 MCBT(R,G,T)          meet auth goals for MAJS in combat arms
 UCBT(R,G,T)          meet auth goals for LTCS in combat arms
 OCBT(R,G,T)          meet auth goals for COLS in combat arms
 FOX(R,G,T)           meet the demand for captains in the combat arms

 LDONOR(D,G,T)        meet auth goals for LTS in donor branches
 CDONOR(D,G,T)        meet auth goals for CPTS in donor branches
 MDONOR(D,G,T)        meet auth goals for MAJS in donor branches
 UDONOR(D,G,T)        meet auth goals for LTCS in donor branches
 ODONOR(D,G,T)        meet auth goals for COLS in donor branches

 LNDB(N,G,T)          meet auth goals for LTS in non detail branches
 CNDB(N,G,T)          meet auth goals for CPTS in non detail branches
 MNDB(N,G,T)          meet auth goals for MAJS in non detail branches
 UNDB(N,G,T)          meet auth goals for LTCS in non detail branches
 ONDB(N,G,T)          meet auth goals for COLS in non detail branches

 COH(T)               distribute the entire cohort

 INV(T)               do not exceed total inventory limit in time t
 BINV(B,T)            meet the total officer authorization in b in time T

 OLDINV(B,K,T)        express inventories in terms of beginning inventory
 NEWINV(B,K,T)        calculate inventory from subsequent accessions

 LYACCA               limit increase in first year's core accessions
 LYACCB               limit decrease in first year's core accessions

 LYDETAXSA            limit increase in current year's branch details
 LYDETAXSB            limit decrease in current year's branch details
```

```
  POSCHANGE(B,T)      limit increase in core accessions from period to period
  NEGCHANGE(B,T)      limit decrease in core accessions from period to period

  PDETDELTA(R,D,T)    limit increase in details from period to period
  NDETDELTA(R,D,T)    limit decrease in details from period to period

  BALANCE(T)          calculate percent of lt auth such that lt inventories balance

  BALRLTS(R,T) balance receiver branch lt accessions

  BALD2LTS(D2,T)      balance two year donor branch lt accessions

  BALD4LTS(D4,T)      balance four year donor branch lt accessions

  LIMRCTL(R,T)        limit receiver control branch accessions
  LIMDCTL(D,T) limit donor control branch accessions;

*--------------------------THE OBJECTIVE FUNCTION----------------------------

OBJ..V =E= SUM(T,DISCOUNT(T)*(SUM(B,BPEN(B)*BSHORT(B,T))
        +SUM((B,G),WU(B,G)*UNDER(B,G,T))
        +SUM(B,OBAL(B,T) + UBAL(B,T))));

*--------------------------GOAL CONSTRAINTS-----------------------------------

LNDB(N,G,T)$(ORD(G) EQ 1)..SUM(L,Y(N,L,T))+UNDER(N,G,T)-SURP(N,G,T) =E= AU(N,G);
CNDB(N,G,T)$(ORD(G) EQ 2)..SUM(C,Y(N,C,T))+UNDER(N,G,T)-SURP(N,G,T) =E= AU(N,G);
MNDB(N,G,T)$(ORD(G) EQ 3)..SUM(M,Y(N,M,T))+UNDER(N,G,T)-SURP(N,G,T) =E= AU(N,G);
UNDB(N,G,T)$(ORD(G) EQ 4)..SUM(U,Y(N,U,T))+UNDER(N,G,T)-SURP(N,G,T) =E= AU(N,G);
ONDB(N,G,T)$(ORD(G) EQ 5)..SUM(O,Y(N,O,T))+UNDER(N,G,T)-SURP(N,G,T) =E= AU(N,G);

LDONOR(D,G,T)$(ORD(G) EQ 1)..SUM(L,Y(D,L,T))+UNDER(D,G,T)-SURP(D,G,T) =E= AU(D,G);
CDONOR(D,G,T)$(ORD(G) EQ2)..SUM(C,Y(D,C,T))+UNDER(D,G,T)-SURP(D,G,T) =E= AU(D,G);
MDONOR(D,G,T)$(ORD(G) EQ3)..SUM(M,Y(D,M,T))+UNDER(D,G,T)-SURP(D,G,T) =E= AU(D,G);
UDONOR(D,G,T)$(ORD(G) EQ 4)..SUM(U,Y(D,U,T))+UNDER(D,G,T)-SURP(D,G,T) =E= AU(D,G);
ODONOR(D,G,T)$(ORD(G) EQ 5)..SUM(O,Y(D,O,T))+UNDER(D,G,T)-SURP(D,G,T) =E= AU(D,G);

LCBT(R,G,T)$(ORD(G) EQ 1)..SUM(L,Y(R,L,T))+UNDER(R,G,T)-SURP(R,G,T) =E= AU(R,G);
CCBT(R,G,T)$(ORD(G) EQ 2)..SUM(C,Y(R,C,T))+UNDER(R,G,T)-SURP(R,G,T) =E= AU(R,G);
MCBT(R,G,T)$(ORD(G) EQ 3)..SUM(M,Y(R,M,T))+UNDER(R,G,T)-SURP(R,G,T) =E= AU(R,G);
UCBT(R,G,T)$(ORD(G) EQ 4)..SUM(U,Y(R,U,T))+UNDER(R,G,T)-SURP(R,G,T) =E= AU(R,G);
OCBT(R,G,T)$(ORD(G) EQ 5)..SUM(O,Y(R,O,T))+UNDER(R,G,T)-SURP(R,G,T) =E= AU(R,G);

BINV(B,T)..      SUM(K,Y(B,K,T)) + BSHORT(B,T) - BOVER(B,T) =E= TOTAL(B,T);

BALRLTS(R,T)..((SUM(L,X(R,T)*RETAIN(R,L))
                 +SUM((L,D2)$(ORD(L) LE 2),DET(R,D2,T)*RETAIN(R,L))
                 +SUM((L,D4)$(ORD(L) LE 4),DET(R,D4,T)*RETAIN(R,L)))/AU(R,'LT'))
                 +UBAL(R,T)-OBAL(R,T)=E=ALPHA(T);
```

```
BALD2LTS(D2,T)..((SUM(L,(X(D2,T)-SUM(R,DET(R,D2,T)))*RETAIN(D2,L))
                    +SUM((L,R)$(ORD(L) GE 3),DET(R,D2,T)*RETAIN(D2,L)))/AU(D2,'LT'))
                    +UBAL(D2,T)-OBAL(D2,T)=E=ALPHA(T);

BALD4LTS(D4,T)..((SUM(L,(X(D4,T)-SUM(R,DET(R,D4,T)))*RETAIN(D4,L)))/AU(D4,'LT'))
                    +UBAL(D4,T)-OBAL(D4,T)=E=ALPHA(T);

*-------------------------HARD CONSTRAINTS-----------------------------------------

COH(T)..SUM(B,X(B,T))  =L= ACCOH(T);

INV(T)..SUM((B,K),Y(B,K,T)) =L= ITOTAL;

OLDINV(B,K,T)$(ORD(T) LT ORD(K))..Y(B,K,T) =E= S(B,T,K-ORD(T))*I(B,K-ORD(T));
NEWINV(B,K,T)$(ORD(T) GE ORD(K))..Y(B,K,T) =E= RETAIN(B,K)*X(B,T-(ORD(K)-1));

BALANCE(T)..ALPHA(T) =E= SUM((B)$(ORD(B) LT 13),X(B,T))
                              *SUM(L,SURVIVE(L))/SUM((B)$(ORD(B) LT 13),AU(B,'LT'));

LIMRCTL(R,T)..X(R,T) + SUM(D,DET(R,D,T)) =L=  LIMIT(R,'UPPER');
LIMDCTL(D,T)..SUM(R,DET(R,D,T)) =L= RHO*X(D,T)*(1-GAMMA(D));

*-------------------------BOUNDARY CONDITIONS---------------------------------

LYACCA(B,T)$(ORD(T) EQ 1).. X(B,T)  =L= UP * LYACC(B);
LYACCB(B,T)$(ORD(T) EQ 1).. X(B,T)  =G= LOW * LYACC(B);

LYDETAXSA(R,D,T)$(ORD(T) EQ 1)..DET(R,D,T) =L= PDELTA * LYDET(R,D);
LYDETAXSB(R,D,T)$(ORD(T) EQ 1)..DET(R,D,T) =G= NDELTA * LYDET(R,D);

POSCHANGE(B,T)$(ORD(T) GT 1).. X(B,T) =L= UP * X(B,T-1);
NEGCHANGE(B,T)$(ORD(T) GT 1).. X(B,T) =G= LOW * X(B,T-1);

PDETDELTA(R,D,T)$(ORD(T) GT 1).. DET(R,D,T) =L= PDELTA*DET(R,D,T-1);
NDETDELTA(R,D,T)$(ORD(T) GT 1).. DET(R,D,T) =G= NDELTA*DET(R,D,T-1);

X.LO(B,T) = LIMIT(B,'LOWER');
X.UP(B,T) = LIMIT(B,'UPPER');


MODEL GOALPGM /ALL/;
SOLVE GOALPGM USING LP MINIMIZING V;


*-------------------------CORE ACCESSION REPORTS-----------------------------
PARAMETER
 AXSRPT(*,*)        officer accession report;
 AXSRPT(B,T) = X.L(B,T);
 AXSRPT('TOTAL',T) = SUM(B,X.L(B,T));
 OPTION AXSRPT:0:1:1;
```

```
PARAMETER
 DELTARPT(*,*)    change in accessions from T-1 to T in branch B;
 DELTARPT(B,T)$(ORD(T) EQ 1) = ((X.L(B,T) - LYACC(B))/LYACC(B))*100;
 DELTARPT(B,T)$(ORD(T) GT 1) = ((X.L(B,T) - X.L(B,T-1))/X.L(B,T-1))*100;

PARAMETER
 STRRPT(*,*,*)      officer strength report;
 STRRPT(T,B,'LT') = SUM(L,Y.L(B,L,T));
 STRRPT(T,B,'CPT')        = SUM(C,Y.L(B,C,T));
 STRRPT(T,B,'MAJ')        = SUM(M,Y.L(B,M,T));
 STRRPT(T,B,'LTC')        = SUM(U,Y.L(B,U,T));
 STRRPT(T,B,'COL')        = SUM(O,Y.L(B,O,T));
 STRRPT(T,B,'TOTAL')   = SUM(K,Y.L(B,K,T));
 STRRPT(T,'TOTAL','LT')= SUM((B,L),Y.L(B,L,T));
 STRRPT(T,'TOTAL','CPT')  = SUM((B,C),Y.L(B,C,T));
 STRRPT(T,'TOTAL','MAJ')  = SUM((B,M),Y.L(B,M,T));
 STRRPT(T,'TOTAL','LTC')  = SUM((B,U),Y.L(B,U,T));
 STRRPT(T,'TOTAL','COL')  = SUM((B,O),Y.L(B,O,T));
 STRRPT(T,'TOTAL','TOTAL')= SUM((B,K),Y.L(B,K,T));
 OPTION STRRPT:0:1:1;

PARAMETER
 FILLRPT(*,*,*)  percent fill of auth in planning year t;
 FILLRPT(T,B,G)$(UNDER.L(B,G,T) GE 0)=((AU(B,G)-(UNDER.L(B,G,T)))/AU(B,G))*100;
 FILLRPT(T,B,G)$(SURP.L(B,G,T) GT 0) =((AU(B,G)+(SURP.L(B,G,T)))/AU(B,G))*100;
 OPTION FILLRPT:0:1:1;

PARAMETER
 BINVRPT(*,*) authorization overages and underages by branch by year;
 BINVRPT(B,T)$(BOVER.L(B,T) GT 0) = BOVER.L(B,T);
 BINVRPT(B,T)$(BSHORT.L(B,T) GT 0) = -BSHORT.L(B,T);
 BINVRPT(B,T)$((BSHORT.L(B,T) EQ 0) AND (BOVER.L(B,T) EQ 0)) = 0;
 OPTION BINVRPT:0:1:1;

PARAMETER
 INVRPT(*,*,*) inventory report;
 INVRPT(T,B,K)              = Y.L(B,K,T);
 INVRPT(T,'TOTAL',K)    = SUM(B,Y.L(B,K,T));
 OPTION INVRPT:0:1:1;
 OPTION Y:0:1:1;

*-------------------------CONTROL BRANCH ACCESSION REPORTS------------------
PARAMETER
  DETREPORT(*,*,*) officer branch detail report;
  DETREPORT(T,R,D)=DET.L(R,D,T);
  DETREPORT(T,'TOTAL',' ')=SUM((R,D),DET.L(R,D,T));
  OPTION DETREPORT:0:1:1;
```

```
PARAMETER
  CTLAXSRPT(*,*)  officer control branch accession report;
  CTLAXSRPT(R,T) = X.L(R,T) + SUM(D, DET.L(R,D,T));
  CTLAXSRPT(D,T) = X.L(D,T) - SUM(R, DET.L(R,D,T));
  CTLAXSRPT(N,T) = X.L(N,T) ;
  CTLAXSRPT('TOTAL',T) = SUM(B,CTLAXSRPT(B,T));
  OPTION CTLAXSRPT:0:1:1;

PARAMETER
  BALAXSRPT(*,*) measure of effectiveness for balanced axs in year T;
  BALAXSRPT('TARGET',T) = ALPHA.L(T);
  BALAXSRPT(R,T) =  ALPHA.L(T)-UBAL.L(R,T)+OBAL.L(R,T);
  BALAXSRPT(D2,T) = ALPHA.L(T)-UBAL.L(D2,T)+OBAL.L(D2,T);
  BALAXSRPT(D4,T) = ALPHA.L(T)-UBAL.L(D4,T)+OBAL.L(D4,T);
  BALAXSRPT(N,T) = SUM(L,X.L(N,T)*RETAIN(N,L))/AU(N,'LT');

*-------------------------SUMMARY REPORT-------------------------------------

PARAMETER
  SUMRPT(*,*,*) basic branch and branch detail summary report;
  SUMRPT(T,R,'CORE') = X.L(R,T);
  SUMRPT(T,'RTOTAL','CORE')= SUM(R,X.L(R,T));
  SUMRPT(T,R,'TWO') = SUM(D2,DET.L(R,D2,T));
  SUMRPT(T,R,'FOUR') = SUM(D4,DET.L(R,D4,T));
  SUMRPT(T,R,'DETAIL') = SUM(D,DET.L(R,D,T));
  SUMRPT(T,'RTOTAL','DETAIL')=SUM(R,SUMRPT(T,R,'DETAIL'));
  SUMRPT(T,R,'CONTROL')= SUMRPT(T,R,'CORE') + SUMRPT(T,R,'DETAIL');
  SUMRPT(T,'RTOTAL','CONTROL')= SUM(R,SUMRPT(T,R,'CONTROL'));
  SUMRPT(T,D,'CORE') = X.L(D,T);
  SUMRPT(T,'DTOTAL ','CORE')= SUM(D,X.L(D,T));
  SUMRPT(T,D2,'TWO') = SUM(R,DET.L(R,D2,T));
  SUMRPT(T,D4,'FOUR') = SUM(R,DET.L(R,D4,T));
  SUMRPT(T,D,'DETAIL') = SUM(R,DET.L(R,D,T));
  SUMRPT(T,'DTOTAL ','DETAIL')=SUM(D,SUMRPT(T,D,'DETAIL'));
  SUMRPT(T,D,'CONTROL')= SUMRPT(T,D,'CORE') - SUM(R,DET.L(R,D,T));
  SUMRPT(T,'DTOTAL ','CONTROL')= SUM(D,SUMRPT(T,D,'CONTROL'));
  SUMRPT(T,N,'CORE') = X.L(N,T);
  SUMRPT(T,'NTOTAL','CORE')= SUM(N,X.L(N,T));
  SUMRPT(T,N,'CONTROL')= SUMRPT(T,N,'CORE');
  SUMRPT(T,'NTOTAL','CONTROL')= SUM(N,SUMRPT(T,N,'CONTROL'));
  SUMRPT(T,'TOTAL','CORE')= SUM(B,X.L(B,T));
  SUMRPT(T,'TOTAL','CONTROL')= SUM(B,SUMRPT(T,B,'CONTROL'));
  OPTION SUMRPT:0:1:1;
```

```
*-------------------------DISPLAY REPORTS------------------------------------

DISPLAY  "0:1:1",AXSRPT;
DISPLAY  "0:1:1",STRRPT;
DISPLAY  "0:1:1",FILLRPT;
DISPLAY  "0:1:1",BINVRPT;
DISPLAY          DELTARPT;
DISPLAY  "0:1:1",INVRPT;
DISPLAY  "0:1:1",Y.L;
DISPLAY          BALAXSRPT;
DISPLAY  "0:1:1",CTLAXSRPT;
DISPLAY  "0:2:1",DETREPORT;
DISPLAY  "0:1:1",SUMRPT;
```

APPENDIX B. DATA

INITIAL INVENTORY $I_{0b}(k)$ (July '95)

(k)	1	2	3	4	5	6	7	8	9	10
IN	504	503	482	387	245	283	298	263	335	275
AR	322	327	302	229	179	208	181	160	157	148
FA	418	468	398	326	273	246	250	234	217	231
AD	177	159	155	120	122	113	90	76	86	64
CM	121	79	91	88	56	45	86	53	72	46
SC	361	306	318	295	219	265	213	216	187	148
FI	43	54	32	39	29	39	39	27	25	14
TC	164	181	151	131	74	102	122	114	100	74
OD	229	247	229	245	140	170	174	155	160	114
QM	244	249	257	200	135	158	205	169	184	169
MI	395	435	350	441	308	315	339	246	268	200
AG	124	126	123	96	86	102	94	102	49	47
AV	327	319	307	344	323	288	267	244	201	192
EN	333	331	289	247	180	160	157	154	133	131
MP	138	124	111	100	86	78	89	99	69	67

(k)	11	12	13	14	15	16	17	18	19	20
IN	247	224	186	194	224	215	218	177	160	213
AR	120	117	89	100	131	135	106	72	103	130
FA	197	183	177	143	167	171	159	154	133	144
AD	83	45	34	41	70	63	71	75	47	54
CM	56	54	38	28	40	40	24	22	22	21
SC	164	122	115	92	103	121	106	81	90	81
FI	25	16	18	22	24	31	23	21	25	34
TC	89	61	35	54	53	39	34	54	46	46
OD	132	92	95	63	83	71	74	84	49	44
QM	142	127	78	77	89	82	92	90	69	62
MI	237	184	153	121	123	132	108	95	121	109
AG	70	51	41	67	87	102	84	66	65	55
AV	158	136	136	151	164	191	145	121	85	114
EN	105	117	97	123	125	106	122	88	92	91
MP	64	52	52	65	65	70	59	46	51	41

INITIAL INVENTORY Cont'd $I_{0b}(k)$ (July '95)

(k)	21	22	23	24	25	26	27	28	29	30
IN	132	90	76	87	62	30	62	30	21	11
AR	46	47	49	40	37	36	23	23	11	3
FA	71	64	61	50	42	43	36	22	15	4
AD	23	23	21	26	15	17	14	4	0	1
CM	4	7	6	14	6	4	6	3	1	2
SC	50	39	38	33	39	33	22	8	2	1
FI	12	16	15	9	8	13	2	4	1	1
TC	25	35	32	22	20	18	14	5	7	3
OD	37	37	38	23	20	32	11	11	12	6
QM	38	39	37	40	31	24	9	8		5
MI	65	46	40	46	33	51	28	10	12	4
AG	28	48	31	21	23	17	16	10	7	0
AV	76	70	56	38	43	51	16	13	8	1
EN	69	52	40	40	29	40	31	20	10	6
MP	26	20	15	22	12	16	8	4	4	5

FY '88 - '89 SURVIVAL RATES S_{bk}

(k)	1	2	3	4	5	6	7	8	9	10
IN	1.0000	0.9868	0.9529	0.8234	0.7097	0.6517	0.6141	0.5873	0.5665	0.5504
AR	0.9999	0.9894	0.9546	0.8052	0.6748	0.6015	0.5662	0.5483	0.5211	0.5028
FA	0.9999	0.9893	0.9545	0.8091	0.6437	0.5721	0.5248	0.4906	0.4685	0.4494
AD	0.9999	0.9859	0.9531	0.8522	0.6647	0.5878	0.5470	0.5174	0.4863	0.4749
CM	0.9999	0.9743	0.9287	0.7964	0.6588	0.6247	0.5838	0.5461	0.5269	0.5090
SC	0.9999	0.9929	0.9654	0.8324	0.6741	0.5992	0.5637	0.5322	0.5046	0.4929
FI	0.9999	0.9654	0.9276	0.8432	0.6826	0.6358	0.6082	0.5643	0.5196	0.5009
TC	0.9999	0.9877	0.9383	0.8058	0.6841	0.6130	0.5894	0.5467	0.5101	0.4747
OD	0.9999	0.9902	0.9516	0.7956	0.6508	0.5924	0.5525	0.5216	0.4992	0.4801
QM	0.9999	0.9885	0.9520	0.8408	0.7250	0.6648	0.6194	0.5765	0.5535	0.5330
MI	0.9999	0.9982	0.9780	0.8765	0.7708	0.7075	0.6700	0.6263	0.5778	0.5498
AG	0.9999	0.9948	0.9711	0.8540	0.7116	0.6250	0.5786	0.5320	0.5104	0.4898
AV	0.9999	0.9926	0.9849	0.9557	0.9055	0.7938	0.7095	0.6527	0.6119	0.5813
EN	0.9999	0.9920	0.9661	0.8216	0.6640	0.5621	0.5006	0.4715	0.4408	0.4241
MP	0.9999	0.9797	0.9091	0.7693	0.6436	0.5959	0.5689	0.5524	0.5221	0.5041

FY '88 - '89 SURVIVAL RATES Cont'd S_{bk}

(k)	11	12	13	14	15	16	17	18	19	20
IN	0.5175	0.4105	0.4035	0.3980	0.3908	0.3839	0.3817	0.3766	0.3690	0.3445
AR	0.4878	0.3734	0.3673	0.3639	0.3610	0.3557	0.3529	0.3491	0.3378	0.3219
FA	0.4389	0.3319	0.3198	0.3160	0.3126	0.3006	0.2964	0.2947	0.2902	0.2715
AD	0.4594	0.3283	0.3204	0.3139	0.3120	0.3031	0.2996	0.2977	0.2977	0.2827
CM	0.4927	0.3599	0.3551	0.3495	0.3495	0.3323	0.3274	0.3274	0.3274	0.3274
SC	0.4793	0.3600	0.3473	0.3408	0.3307	0.3237	0.3181	0.3160	0.3049	0.2857
FI	0.5009	0.3866	0.3717	0.3641	0.3497	0.3375	0.3283	0.3220	0.3171	0.2944
TC	0.4505	0.3576	0.3491	0.3440	0.3417	0.3376	0.3309	0.3283	0.3201	0.3112
OD	0.4685	0.3354	0.3226	0.3169	0.3107	0.3053	0.2993	0.2993	0.2897	0.2647
QM	0.5189	0.3740	0.3648	0.3579	0.3535	0.3506	0.3472	0.3429	0.3287	0.3061
MI	0.5312	0.4129	0.3978	0.3924	0.3878	0.3797	0.3753	0.3742	0.3654	0.3372
AG	0.4732	0.3661	0.3548	0.3479	0.3449	0.3370	0.3283	0.3269	0.3243	0.3064
AV	0.5614	0.4369	0.4265	0.4212	0.4160	0.4072	0.4013	0.3957	0.3761	0.3370
EN	0.4055	0.3280	0.3185	0.3173	0.3173	0.3123	0.3098	0.3098	0.3038	0.2813
MP	0.4964	0.4031	0.4008	0.3954	0.3926	0.3897	0.3846	0.3765	0.3666	0.3243

(k)	21	22	23	24	25	26	27	28	29	30
IN	0.2900	0.2533	0.2214	0.1770	0.1462	0.1205	0.0845	0.0634	0.0287	0.0172
AR	0.2860	0.2469	0.2210	0.1926	0.1641	0.1409	0.1083	0.0759	0.0562	0.0402
FA	0.2285	0.1886	0.1681	0.1366	0.1248	0.1066	0.0833	0.0598	0.0344	0.0207
AD	0.2363	0.1937	0.1677	0.1274	0.1084	0.0903	0.0661	0.0466	0.0329	0.0200
CM	0.2873	0.2713	0.2437	0.1968	0.1657	0.1450	0.1044	0.0714	0.0524	0.0262
SC	0.2071	0.1611	0.1301	0.1061	0.0923	0.0800	0.0590	0.0302	0.0206	0.0099
FI	0.2346	0.1631	0.1254	0.0971	0.0825	0.0642	0.0453	0.0415	0.0208	0.0052
TC	0.2697	0.2251	0.1814	0.1421	0.1111	0.0937	0.0803	0.0619	0.0263	0.0179
OD	0.2165	0.1784	0.1545	0.1234	0.1044	0.0923	0.0702	0.0465	0.0257	0.0117
QM	0.2530	0.2191	0.1894	0.1449	0.1207	0.1024	0.0795	0.0499	0.0261	0.0153
MI	0.2750	0.2288	0.1943	0.1600	0.1396	0.1224	0.0930	0.0668	0.0340	0.0238
AG	0.2411	0.1913	0.1603	0.1380	0.1230	0.0970	0.0737	0.0465	0.0272	0.0104
AV	0.2658	0.2125	0.1667	0.1290	0.1065	0.0891	0.0705	0.0480	0.0299	0.0175
EN	0.2269	0.1883	0.1547	0.1375	0.1173	0.0962	0.0667	0.0470	0.0319	0.0206
MP	0.2433	0.2141	0.1758	0.1460	0.1207	0.1073	0.0816	0.0489	0.0309	0.0174

FY ' 88 - 89 ARMY AVERAGE SURVIVAL RATES

(k) 1	1.00000
2	0.98947
3	0.97177
4	0.90367
5	0.75309
6	0.65522
7	0.59963
8	0.55646
9	0.52063
10	0.49596
11	0.47152
12	0.40061
13	0.36520
14	0.35276
15	0.34191
16	0.33154
17	0.32494
18	0.32108
19	0.31604
20	0.30197
21	0.25364
22	0.20944
23	0.17722
24	0.14394
25	0.11661
26	0.09662
27	0.07349
28	0.05134
29	0.03039
30	0.01585

FY '95 BRANCH DETAIL PLAN
LYDET(r,d)

	IN	AR	FA	AD	CM
SC	57	36	30	19	19
FI	1	5	8	2	2
TC	19	13	13	7	6
OD	22	16	12	8	8
QM	32	22	20	10	12
MI	52	35	54	19	13
AG	18	11	17	6	4

FY '95 CORE ACCESSIONS
LYACC(b)

IN	504
AR	322
FA	418
AD	177
CM	121
SC	360
FI	43
TC	165
OD	230
QM	245
MI	396
AG	124
AV	327
EN	333
MP	138

FY '95 AUTHORIZATIONS
ADJUSTED NOF '97

	$AU_{bg}(t)$					$TOTAL_b(t)$
	LT	CPT	MAJ	LTC	COL	TOTAL
IN	1936	2044	793	466	192	5431
AR	1028	1305	526	230	109	3198
FA	1251	1692	760	335	124	4162
AD	492	718	356	207	55	1828
CM	393	489	240	142	38	1302
SC	666	1722	881	449	209	3927
FI	30	160	117	76	24	407
TC	274	517	442	308	122	1663
OD	388	944	574	308	102	2316
QM	292	997	605	343	108	2345
MI	505	2041	1027	478	175	4226
AG	96	499	332	177	47	1151
AV	798	1584	726	343	81	3532
EN	550	1376	670	475	276	3347
MP	292	580	320	213	63	1468

PENALTY WEIGHT $W_{bg}(t)$

	LT	CPT	MAJ	LTC	COL
IN	36	50	15	9	4
AR	32	50	16	7	3
FA	30	50	18	8	3
AD	27	50	19	11	3
CM	30	50	18	11	3
SC	17	44	22	11	5
FI	7	39	29	19	6
TC	16	31	27	19	7
OD	17	41	25	13	4
QM	12	43	26	15	5
MI	12	48	24	11	4
AG	8	43	29	15	4
AV	23	45	21	10	2
EN	16	41	20	14	8
MP	20	40	22	15	4

FRACTION OF FEMALES γ_d

SC	0.21
FI	0.25
TC	0.22
OD	0.20
QM	0.19
MI	0.19
AG	0.48

PARAMETERS

ACCOH(t)	3900
ITOTAL(t)	51165
UP	1.2
LOW	0.9
Δ^+	1.33
Δ^-	0.67
ρ	0.7
r	0.1

LIMITS ON YEARS OF SERVICE IN EACH GRADE

	LT	CPT	MAJ	LTC	COL
UPPER	1	5	12	17	23
LOWER	4	11	16	22	30

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